

NOTES - ON - THE - SCIENCE - AND
ART - OF - EDUCATION • NOETLING

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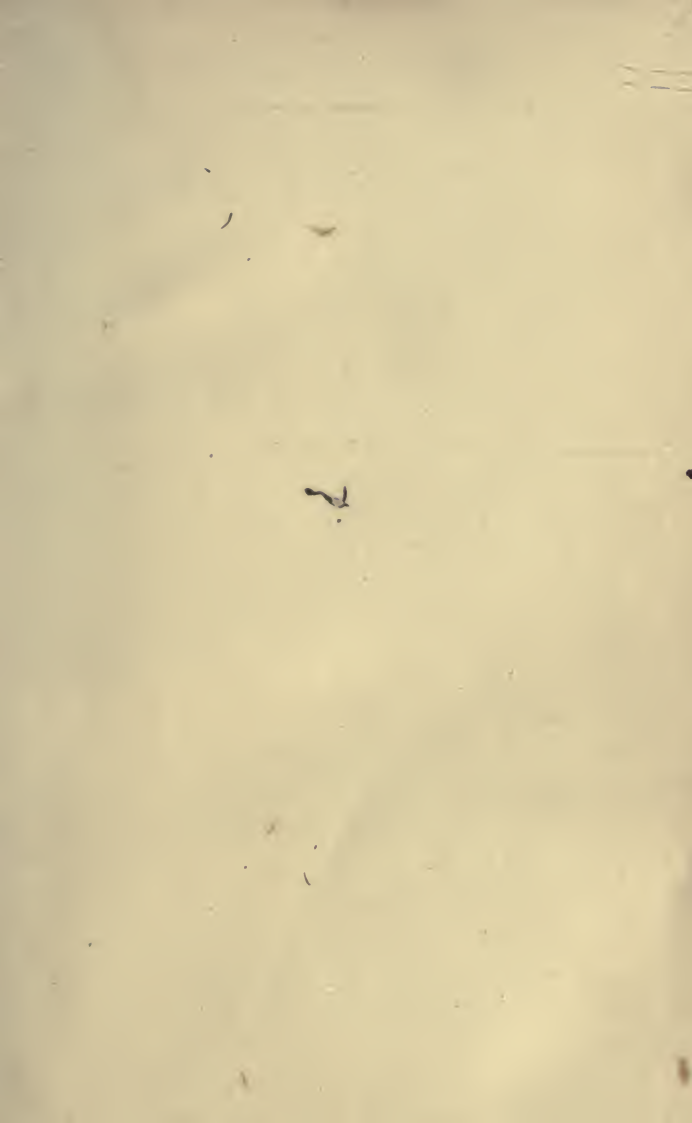


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NOTES
ON THE
SCIENCE AND ART
OF
EDUCATION.

BY
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PREFACE.

I HAVE for some years dictated my instructions on the science and art of teaching, in the form of notes, suggestions, and hints, to the junior class of this school; but a desire has been expressed by students and others to have the work in a more convenient shape for preservation and use, and to enable the many teachers who do not have the advantages of normal instruction to avail themselves of its helps. In compliance with this desire I have prepared the *notes* for publication: . They do not, however, constitute a methodical or a complete treatise upon pedagogics, but only thoughts and suggestions for prospective teachers and for beginners in school-room work.

Every subject has been treated with as much fullness, as well as brevity, as my experience has shown necessary. To subjects in which beginners need most help I have given more space than to others; this accounts for the disproportion in the number of pages devoted to different subjects.

The matter will be found in harmony with the best in education and teaching, and presented, it is hoped, with a sufficiency of explanation to make it intelligible to beginners.

Repetitions occur here and there, wherever it is believed they will be serviceable to those for whom the book is intended.

WM. NOETLING.

STATE NORMAL SCHOOL,
BLOOMSBURG, PA.,
Dec. 19, 1894.



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NOTES ON THE SCIENCE AND ART OF EDUCATION.

INTRODUCTORY CONSIDERATIONS.

1. The science of education embraces the principles, laws, or knowledge in accordance with which the education of a child must be carried on ; and the art, the carrying on of the process. In other words, the science is the knowing, and the art the doing.

2. To conduct the education of a child intelligently and successfully, the teacher must possess a thorough knowledge of its constitution, physical and mental.

3. Understanding the physical constitution implies a knowledge of its mechanism, the function each organ performs, and the relation the various organs bear to one another.

4. Enjoyment is one of the most important elements in our existence upon earth ; and this depends upon health—the health of the body and of the mind. Preserving health, or securing it if not possessed, is therefore one of the first things, if not altogether the first, in the education of a child.

Part I.

THE CARE OF THE BODY.

5. To preserve the body in a healthy state, it must have a proper supply, as fast as needed, of the elements that enter into its composition ; it must be kept clean, properly clothed, must have rest and exercise; and be surrounded by suitable light and proper conditions of atmosphere.

6. The body is composed of a number of kinds of material, and the quantity of each necessary at any time to preserve the health and strength of every organ and power depends upon the kind of activity in which the person is engaged, or, in other words, upon the amount of activity of each organ and power. For example, if the muscles chiefly are exercised, the material of which they are formed must be supplied in sufficient quantity to make up for their waste—the amount destroyed by use ; if the bones, the material of which they are made must be supplied in sufficient quantity to make up for their loss by use ; and if the brain and nerves, the kind of material that builds them up must predominate in the nourishment taken.

7. The proper nourishment of the body does not, however, alone depend upon the quantity of any kind of food taken, but more, perhaps, upon the quality and upon the preparation it has received.

8. Another important condition of food is variety. An unvaried diet destroys both appetite and health. Walker, in his *Physiology*, says : “ The system craves a varied diet, and living for a length of time on even an abundance of food, if it be unvaried from day to day, will generally result

in loss of appetite and in disease. * * * When a variety of articles cannot be obtained, varied methods of preparing and cooking the limited supply should be resorted to. 'Good cookery means economy; bad cookery means waste.'

"On the other hand, however, there may be such a thing as too great a variety, and this also will destroy the appetite."

"It is a dictum of mental as well as physical hygiene that it is far better to stint one's self along any other line rather than deprive ourselves of food of needed quality and quantity. I say *stint*, for it is *not* economy. Poor food means poor blood and not enough of it, and this in turn means a brain starving for oxygen. Such a brain is always a weary brain, slow to respond and erratic in its activities; and this fatigued, poisoned brain can never sustain mental processes of high character or strict integrity. Therefore, I say, that in treating of the reciprocal influence that obtains between body and mind, the question of diet is one of special importance to those having the care of children, and should be discussed at great length by educators in order that it may receive in every quarter the attention it so richly deserves."—Krohn, *Practical Lessons in Psychology*.

9. Exercise, too, at proper times, and suitable in kind and amount, is a necessity to health and comfort. But exercise of any part of the body tears it down—in other words, gradually wears it out, consumes its tissues; and if this process is continued too long or faster than new ones are formed to take the places of those that have been destroyed, pain, in form of weariness or fatigue—nature's warning—ensues, and is a sign that the safety-point has been passed and that rebuilding is necessary.

10. Rebuilding of brain requires more time than that of muscle or of bone; brain-workers, therefore, demand more sleep than those who chiefly exercise their muscles. But sleep should be sound, unbroken. Students, when permitted to do so, frequently study at hours of the night when they should sleep, and afterwards try to sleep when, from exhaustion or worry, they are unable to do so. But wake-



fulness, when the physical system needs repairs through rest, implies some kind of functional disorder, and undoubtedly is a premonition of brain-exhaustion. Dr. J. L. Corning, an authority on cerebral diseases, says : " Derangement in the function of sleep is an infallible sign that the proper relation between waste and repair of brain-tissue no longer exists; and that, unless the undue expenditure of brain-force be made to cease, cerebral bankruptcy is impending. * * * The injury produced upon the thought and emotional centres by a high degree of worry, conjoined with undue intellection, it is almost impossible to overestimate; indeed, a very large percentage of cases of brain-exhaustion is directly traceable to this baneful combination of causes."

11. When sleeplessness, headache, loss of appetite, languor on rising in the morning, and general debility manifest themselves, and drugging begins, the danger-signals are clearly in view, and, if not heeded, a wreck is sure to follow. (For a fuller treatment on securing and preserving health, see Walker's *Physiology* and Martin's *The Human Body*.)



Part III.

THE MIND.

CHAPTER I.—THE INTELLECT.

12. Method of Studying It.—What mind is we do not know, in fact cannot know ; for it is not a material thing, a thing that can be cognized by the senses. Like electricity and magnetism, it can be studied only through its manifestations or effects. This method of learning is that of inference, inferring causes from their effects—learning the promptings of the mind by the appearances and muscular activities that follow them, and that seem to be their effects or that we infer to be their effects. Mental states and activities must therefore be studied inductively. Generalizations must be cautiously made, and no conclusion hastily drawn. Facts, sufficient in number, must be observed, noted, and classified, before a valid conclusion can safely be reached. Nor will one person's facts serve any purpose for those of another ; each must make his own observations, classifications, and draw his own conclusions. This is what each *must* learn to do in order to become a student of mental phenomena. Neither the teacher nor any one else can do it for him ; all the help that any one can give him is to show him how to study himself and others, and how to test his conclusion.

13. The Body the Servant of the Mind.—The mind manifests itself through the body, performs all its observable

or visible work through it ; and one of the objects of education is the training of the body to become the obedient and skilful servant of the mind.

REMARK.—Under no circumstances must the mind become the mere servant of the body, or the body be allowed complete control of the mind.

14. Forms of the Material World and How Cognized.

—The material world presents itself to us in six different forms: 1. Colors and figures ; 2. Sounds ; 3. Heat and cold ; 4. Hardness and softness, smoothness and roughness ; 5. Tastes ; 6. Smells. And the organs through which these forms are cognized are the senses, the feelers—seeing, hearing, touching, tasting, and smelling. It is, however, the mind that sees, hears, touches, tastes, and smells, and not the organs which it uses to do so.

15. When an object affects a sense-organ, the nerves connected with the organ convey the impression to the mind ; in other words, make the mind conscious of it. If the object is of sufficient interest or importance at the time, the mind observes it and takes, so to speak, an impression or image of it. This impression is termed a percept.

REMARK.—A percept remains before the mind, or in consciousness, only as long as the object which gives rise to it affects the sense-organ.

16. **Attention.**—In order that the mind may be in the most favorable state for the reception of impressions, certain conditions are necessary. One of the most important of these is attention, or the concentration of the mind upon the object examined or subject studied.

17. For educational purposes, attention may be divided into two kinds, apparent and real ; the former being only the appearance of attention, the latter the reality.

18. Real attention may be either attracted or directed, and divided or undivided.

REMARK.—Attracted attention is also called non-voluntary (without an effort of the will), and directed, voluntary (by an effort of the will).

19. Apparent and divided attention, having no value in education, need no further consideration here.

20. Attracted attention is that which is given from interest or novelty in the subject examined or studied, and depends upon the teacher's ability to make the subject of instruction interesting or attractive.

REMARK.—Attracted attention is the only kind that can be expected from children.

21. Directed, or voluntary, attention is that which comes from an effort of the person giving it. This kind can be expected from persons of sufficient age and judgment to appreciate the value of subjects of study, but not from children. Voluntary attention must, however, ultimately change to non-voluntary. If it fails to do this, the teaching is defective—a failure.

22. Attention cannot be given equally well under all circumstances. Its most successful efforts depend upon certain conditions ; the most important of which are : (1) good health, (2) good light, (3) pure air, (4) proper temperature, (5) comfortable seats, (6) absence of distracting objects, (7) proper position, and (8) close classification.

23. Conditions of attention, however important, do not imply attention ; they imply simply that everything necessary, so far as the pupils and their surroundings are concerned, has been supplied. The next thing to do is to secure attention, and this not all teachers can do with equal success. As a requisite, to begin with, the teacher must have the confidence of his pupils. He should be : (1) master of the subjects he teaches ; (2) unhampered by the use of a text-book during recitation ; (3) cheerful ; (4) interested ; (5) in earnest. He should begin where the children's knowledge ends ; and should excite curiosity.

REMARK.—The teacher should not abruptly pass from one subject to another, but should by gradual steps lead his pupils to it. Inattention is frequently caused by abrupt transitions.

24. Attention must not only be secured, but must be continued or held. The following suggestions, if carried out in the proper spirit, not merely in a mechanical manner, will aid the teacher in keeping the attention of his pupils : 1. Do not attempt to teach anything that is beyond the ability of the pupils ; 2. Keep curiosity aroused ; 3. Let the age of the pupils determine the length of the lesson ; 4. Let the advancement determine the subject and the lesson ; 5. In teaching children, sense-wholes should be taught before their parts ; 6. Go from the known to its related unknown ; 7. As a general rule, tell nothing to pupils which they can find out themselves or be led to find out ; 8. Vary your mode of instruction, your mode of presenting subjects ; 9. As far as possible, give every pupil a share in the recitation ; 10. Use illustrations and apparatus ; 11. Do not teach longer than you have the attention of your pupils ; 12. Utilize the instincts—activity, curiosity, imitation, etc.

REMARK ON CONDITION 10 OF THE FOREGOING.—If you have apparatus, teach with it. Requiring pupils to prepare lessons from imperfect descriptions or poorly-made illustrations or pictures, when a school has the apparatus with which the subject may be studied, is inexcusable.

REMARK ON SECURING AND HOLDING ATTENTION.—To secure and to hold attention, something new must be presented, or the mode of presentation must be new. No one can give continued attention to that which yields nothing.

25. Like the other powers of the mind, attention may be cultivated. The following are some of the means that may be used for this purpose : 1. Read or relate something of interest and value to the pupils, to be reproduced by them ; 2. Require them to reproduce from memory a conversation, lecture, address ; 3. Require the reproduction of a problem read to them or by them ; 4. Require the reproduction of a paragraph, article, or selection read by them. ;

REMARK.—Special periods for the purpose of cultivating the power of attention are not to be recommended ; for the same ends may generally be attained in the regular classes with the daily work.

26. Attention Depending upon Pupils' State of Mind.

—Besides the foregoing conditions of attention, there are others which, on account of their relation to successful school-work, are of sufficient importance to merit separate consideration. 1. The pupils may be tired ; 2. Their minds may be occupied with something to them of more interest than that which the teacher is endeavoring to introduce ; 3. They may be in a so-called neutral or indifferent state ; 4. They may be in a state of expectancy—waiting, with interest, for the lesson to be commenced.

In the first case, they should either be excused from further work, or given some such exercises as the making of forms of different kinds of material ; work in which they can be interested and which will not tire them.

The second state may be one of antagonism or opposition, one in which they refuse to take part or interest in the subject which the teacher desires to introduce. In this case the only remedy is to prepare the way, step by step, by the introduction of something which shall cause them to forget their former thoughts and thus prepare them for the introduction of the lesson for the period.

The third condition needs the same kind of preparatory treatment as the second, the only difference being that, as a general thing, it requires less effort on the part of the teacher.

In the fourth condition the pupils are eagerly waiting for the lesson to begin, and consequently need no preparation for it.

27. Perception—Observation—Reflection.—As already stated, the impression which an object makes upon the mind through the senses is called a percept ; in other words,

a percept is a mental construction of an external object—a construction in the mind by the mind itself.

A percept is a product, and the operation or act that gives rise to it is perception.

Percepts are of two kinds, original and acquired.

REMARK 1.—Some psychologists apply the term *perception* to a complete mental picture of an external object, and percept to a single element of it obtained through one of the senses.

REMARK 2.—Perceptive knowledge, or that obtained from the direct or immediate apprehension of an object, is also called presentative knowledge, and its revival or representation from memory, representative knowledge.

28. Our knowledge begins with experience ; it depends, consequently, upon the completeness and clearness of our percepts ; and the quality of our percepts depends upon observation—upon attention to all the particulars or characteristics of an object. Careful observation therefore lies at the foundation of education.

29. Not only can the mind observe the outer, the world of matter, but it can also turn itself in upon itself, so to speak, and scan its own states and activities. This process—observing what the mind itself is doing—is called introspection, reflection ; sometimes, internal perception.

30. **Memory.**—Memory (figuratively speaking) is the mind's treasure-house, the power of preserving and recalling the facts of consciousness. When attention is withdrawn from an object, its percept passes from consciousness to the memory.

31. For the convenience of study, memory may be considered under two distinct heads, retention and recollection. Retention is the mind's power of holding or preserving the facts of consciousness, and recollection that of bringing them forth when they are wanted.

REMARK.—James Mark Baldwin, in his *Elements of Psychology*, says :

"In considering the entire mental function which we call

memory, we find that it involves several factors or stages, which are sometimes treated as distinct operations, but may properly be considered, as we find them, together. Together they constitute a chain of events whereby the mental life of the past is retained and utilized in the present. First, there is the permanent possibility of the revival of a past experience when its first circumstances are repeated; this is called *Retention*. Next, there is the actual return of the image to consciousness: *Reproduction*. Third, this image is known as having already been presented in our past experience: *Recognition*. And finally there is, in most cases, an immediate reference to the exact past time of its first experience: *Localization in time*. These, taken together, constitute a finished act of memory. Accordingly, memory may be defined as a *mental revival of a conscious experience*."

32. As before remarked, when an object of perception no longer affects or stimulates a sense-organ, no percept of it is in consciousness; recollection therefore does not bring percepts before the mind, but transcripts, mental representations, images, ideas, or concepts of them.

REMARK 1.—The term concept is preferable to that of idea for recalled mental pictures, because idea is used in so many senses as to lead to confusion in the minds of students of mental phenomena. Trench says: "The word idea is, perhaps, the worst case in the English language. One person, for example, has an idea that the train has started, another had no idea that the dinner was so bad."

REMARK 2.—Conception is a constructing process, and its products are concepts.

REMARK 3.—All forms in which past experiences may be recalled and brought before the mind will in these pages be considered as representations or mental pictures, and all mental pictures, except percepts, as concepts.

33. The mind's power to retain facts depends greatly upon the state or condition in which it is when it receives them. Among the most important conditions of retention are the following: 1. Healthy and fresh state of body and mind; 2. Undivided attention; 3. Thorough, clear, and distinct comprehension; 4. Lively and sincere interest; 5. Determination, or force of will; 6. Repetition; 7. Sufficient time for the impression to be made.



34 An examination of the foregoing conditions of retention reveals the fact that retention is not an active power of the mind, and that it is cultivated only through the activity of the other powers ; its highest degree of success depends, therefore, upon the perfection of the activity of the other powers.

35. Mental images play an important part in remembering. Kay, in his book on the memory (page 208, etc.), says :

“ The subject of mental images is one that has hitherto received but little attention, and yet it is one of the deepest interest, and calculated to throw light upon many obscure mental phenomena. Whenever a sensation or an idea is presented to the mind, a mental image or conception of it must be formed in order to its being perceived or understood. In proportion to the clearness and distinctness of the image will be the understanding of it by the mind, and the hold taken of it by the memory.

“ As there are different kinds of sensations and different classes of ideas, there exists a like variety among mental images ; and some minds excel in some, others in other. Thus, some may excel in the formation of visual images, others of auditory ones. The former will remember best those things that are presented to the eye, and of which they can form visual images ; the latter, such as are addressed to the ear, and form auditory ones. The former will take in and remember what they read, the latter what they hear ; the one will learn a language most easily by the eye, from books ; the other by the ear, from conversation. Some, in listening to a discourse, image every word they hear as it appears to the eye ; while others, with the auditory faculty largely developed, will image what they read as if it were addressed to the ear. Others, again, in reading or in listening to a discourse, will attend only to the sense or meaning, and form sense-images. These can give the substance of what they have read or heard with great accuracy, though they may not perhaps be able to recall any of the words. In each case it is of importance to ascertain in what direction the image-forming power of the mind chiefly lies.

“ Further, not only are there images of the eye and ear, and of the other senses, but there are also images of muscular movements, as of the tongue and hand. Some may not remember much of what they see or hear, but remember readily what they say or do. Hence some children learn best by repeating aloud, others by writing down what they wish to remem-

ber. Most persons have probably observed, in writing a word in regard to the spelling of which they are sometimes in doubt, that if they write it at once, without thinking about it, they usually spell it correctly; but if they doubt and hesitate and think, they become uncertain, and most probably spell it wrong. The reason is that the mental image which directs the hand is, in this instance, a surer guide than that furnished by the intellect. In such cases the more the mind is engaged in thought the less able is it to listen to those inner promptings of our nature—the muscular images of past movements, on which so much that is finest and most delicate in our action depends.

“ But not only are there in the mind mental images of sensible objects, and of muscular movements, of what we feel and what we do, but every thought, however abstract or apparently disconnected from sensible objects, has its image in the mind. We can only conceive an abstraction by having an image of it. The abstract idea of a triangle, which is not any particular triangle, but represents the properties common to all triangles, has as much its image in the mind as any individual triangle that may have been before it. Further, we must regard each abstract idea as having a physical state corresponding to it; and hence we can localize abstract ideas and recall the occasions when they were present.”

36. Not only must facts be retained in the mind for future use, but they must be recalled when wanted. Minds differ much in their recalling power; some readily recall one kind of facts, others another kind; but, in either case, the ease with which they can be brought forth depends upon the manner in which they were stored away in the memory. If they are associated, or presented to the mind, in a systematic, related manner, they will be returned in the same manner, every link in the chain of ideas suggesting its neighbor.

37. Concepts or ideas may for educational purposes be associated in four ways: 1. By contiguity; 2. By similarity; 3. By contrast; 4. By cause and effect. Contiguity means adjoining, contact; similarity, likeness, resemblance; contrast, dissimilarity; cause, that which produces a change, an effect; and effect, that which has been brought about or produced by a cause.

REMARK.—Other modes of association besides the foregoing are sometimes given by writers on psychology, but as they are of no service to teachers, they are here omitted.

38. The underlying principle of contiguous association is that concepts or experiences which occur together or in immediate succession afterwards tend to revive one another. Observation, too, seems to teach that the mind integrates or “completes any process upon which it enters, if it has performed the same process before.”

39. Contiguity may be in time or in place. One thing suggests another that appeared before the mind with it or immediately before or after it. Things that were before the mind either at the same time or in successive time, or that were before it together in space (place), are revived together or have a tendency to reappear together.

REMARK.—A number of dissimilar objects may be placed in a fixed order and remembered or recalled in that order, by associating them in the mind invariably in the same order.

40. An object or concept will bring before the mind others that bear a resemblance or an analogy to it. When a relation of some kind can be discovered among objects or concepts, and this relation is made the basis of a systematic arrangement of the objects or concepts, so that any one in the series, in consequence of the relation, will suggest the next, the recalling process is much easier than when the association is arbitrary, mechanical, without relation.

41. By the association of similars we are enabled to form classes of objects or concepts, on account of their resemblance in form, material, or quality, and to recall them. By their resemblances, also, we trace the relations of words and thus discover their meanings. The study of languages is greatly shortened by the association of similars.

42. For the purpose of firmly fixing anything in the mind and readily recalling it when it is wanted, association by

contrast is more serviceable than association by resemblance or analogy. Tate, in his *Philosophy of Education*, says : "Associations of resemblance are rarely so vivid as those of contrast ; and hence it follows that scenes or events which are in contrast with each other are more likely to be remembered than those which have a resemblance. Contrast, like light and shadow, makes the objects more prominent."

REMARK.—Only such things as have some points of similarity can be contrasted. Those which cannot be compared cannot properly be contrasted.

43. Landon, in his *School Management*, says : "The natural relationship which links ideas by means of cause and effect renders them eminently suggestive of one another ; and where the connection exists, to fix it clearly in the mind is one of the most powerful means of association at our disposal. It is especially valuable in lessons from the physical or natural sciences ; and should be much more generally employed in the teaching of such subjects as history than it appears to be."

Tate, the author above quoted, says : "The minds of children are so constituted that they most readily remember effects in connection with their causes ; for example, they readily associate the light of day with the presence of the sun ; storms with winds and clouds ; the heat of summer with the long days of sunshine ; the improvement of the mind with application to study ; misery with crime, and happiness with virtue ; and so on. Associations of this kind are most interesting and instructive ; one idea becomes the nucleus of a whole series, and idea becomes so linked with idea that we are enabled to form a continuous chain of them. Thus we readily remember the following chain of associations : Rain falls from the clouds ; the clouds are chiefly formed by winds and mountains ; the cold on the tops of the mountains condenses the moisture

in the air, and thus clouds are formed; the cold on the tops of the mountains is caused by the thinness of the air, etc.; thin air is colder than dense air, because it has greater capacity for heat, and so on."

44. From the foregoing statements concerning the linking of concepts it will readily be observed that there are really but two forms, or modes, of association, arbitrary or mechanical, and philosophical or logical. Association by contiguity is arbitrary or mechanical, and the association of concepts by such a relationship that one calls into consciousness another bearing a resemblance of some kind to it, or belonging to the same logical connection, is properly termed philosophical or logical.

45. Association by contiguity, of necessity, has its place in the acquisition of knowledge; but philosophical or logical relationships are more valuable in the development of the mind, because they call into activity the higher mental powers. No philosophical or logical bond can be discovered without thought.

REMARK.—Children and the illiterate depend chiefly upon mechanical associations; older people, and especially the educated, use philosophical or logical relationships.

46. There is only one mode of acquiring facility in the suggesting, or recalling, process, and that is intelligent, persistent practice.

47. The following are some of the subjects that may be taught by resemblance and contrast.

A. RESEMBLANCE. — *a. Geography.* — North and South America; South America and Africa; California and Spain or France; Italy and India; Australia and Cuba; North America and Europe; Pennsylvania and Missouri or West Virginia; New York and Boston or Chicago; Baltimore and Philadelphia; Richmond and Albany; Atlantic Ocean and Pacific Ocean; products of East Indies and West

Indies ; climate and products of southern U. S. and those of India ; etc.

b. History.—Settlement of Massachusetts and of Virginia, of New York and of Massachusetts, of Pennsylvania and of Maryland, of Ohio and of Connecticut, of California and of Kansas, of New Jersey and of Georgia ; Washington's and Jefferson's administrations, Washington's and Lincoln's ; Columbus and Captain Cook, Lincoln and Garfield, Grant and Napoleon, Bacon and Newton, Pestalozzi and Horace Mann, Bryant and Longfellow, Daniel Webster and John C. Calhoun, Horace Greeley and John Bright ; etc.

B. CONTRAST.—*a. Geography.*—The Old World and the New ; the two hemispheres (eastern and western) ; London and New York ; Spain and Italy ; coast of Europe and of North America ; eastern coast of North America and western coast of same ; valley of the Mississippi and that of the St. Lawrence ; North and South America ; Africa and South America ; Philadelphia and Chicago ; etc.

b. History.—Settlement of Jamestown and of Massachusetts Bay Colony ; colonists of Virginia and of Massachusetts ; Dutch of New York and Quakers of New Jersey and Pennsylvania ; colonists of Maryland and of Virginia ; Washington and Jefferson ; Clay and Webster ; etc.

c. Physical Geography.—Surface of Pennsylvania and of New York ; climate of Pennsylvania and of New York or Massachusetts ; products of Pennsylvania and of Massachusetts, Maine, or Virginia ; climate and productions of North America and of South America ; valley of the Danube and of the Mississippi ; climate and productions of Russia and of France, of Ireland and of Australia, of Arabia and of India, of Siberia and of British America, of Southern Africa and of the southern part of South America, of North Temperate and South Temperate Zone ; etc.

C. MISCELLANEOUS.—Octagon and circle ; degrees of hardness and softness ; resemblances of color ; light and

heat ; cylinder, cone, and sphere ; forms of letters of the alphabet ; spelling of words of similar forms and sounds ; etc.

48. The learning of words depends upon association. The child associates the name of a word with the form of the word, and either one of them suggests the other. So also in acquiring knowledge are the names of things associated with the things themselves, and thus remembered and recalled.

REMARK.—The work of the primary teacher consists largely in helping the children to form permanent associations.

49. Repetition and reviews constitute the main reliance for forming lasting associations ; but the repetitions and reviews must not be allowed to become monotonous. Pleasurable emotions, or interest, can be excited and kept up in no other way than by variety and novelty.

50. If memory is found weak in any particular direction,—in recalling names, dates, etc., for example,—and any one's duties demand much work of this kind, the only remedy is daily exercises of the kind required. It is wonderful to what an extent the memory can be trained in any special direction by persistent efforts.

51. Sully says : “Committing anything to memory is a severe demand on the brain-energies, and should, so far as possible, be relegated to the hours of greatest vigor and freshness. The morning is the right time for learning. In addition to selecting the best time, every resource should be used to make the subject as interesting as possible.”

REMARK.—A ready memory is undoubtedly an invaluable possession, yet it needs to be carefully kept within its legitimate bounds ; it should not be permitted to supplant any of the other powers—perception, imagination, judgment, etc.

52. Imagination.—Imagination is the power which forms or constructs mental images. It is divided into constructive, reconstructive, and productive invention.

53. Construction may take place from observation or from description—from what we ourselves observe or from what we learn from others. Perceiving is a constructing process ; it is forming images in the mind ; and though not generally called imagination, it deserves as much that name as any other picturing process performed by the same power.

54. In the study of all branches of knowledge in which, in the absence of the objects or phenomena, mental pictures are required, the imagination constructs and paints them. In the study of geography and history the use of the constructive imagination is indispensable.

55. Reconstruction takes place in recollection or from memory ; it is rebuilding that which at some previous time had been built or formed ; it is constructing according to a former pattern or model.

56. Reconstruction is employed in all recitation. The pupil reconstructs his former constructions, and the teacher and the class observe whether they are correct or reasonable.

REMARK.—The criticisms made by the teacher and the pupils should enable the one who recites to correct and complete his constructions. Simply pointing out errors, without indicating how they may be corrected, has some value ; but pointing them out in such a way as shall enable the student to see them and correct them is the proper way to make the corrections. Criticisms should always be helps, not hindrances.

57. Production, or invention, has reference to original combinations or constructions. The ability to make these cannot be directly taught ; it can only be encouraged, not learned ; it can only be acquired by those who have the talents and the patience—by continued study and practice. One who succeeds in the highest forms of original construction is said to be a genius.

58. The teacher should not only allow his pupils the exercise of originality, but should encourage it whenever it is practicable to do so. Throwing them as much as possible

upon their own resources is one of the best ways of doing this.

59. Currie, in his *Principles and Practice of Common-School Education*, says :

"Observation is limited by very narrow boundaries of time and space; to whatever extent we pass these, it must be on the wings of imagination. Accordingly, descriptions of natural scenery and scenes from life, real or ideal, are the field in which this mode of intelligence must be exercised; and both are very rich in materials.

"When the pupil has observed the elements of the landscape at home, he is required to carry these abroad and, by modification, interchange, and amplification, to construct another landscape there; the hill, rivulet, meadow, and wood of his own native district become the snow-clad peak or volcano, the mighty river, the far-spreading desert or cultivated plain, the trackless forest, of other lands; from the summer's heat and winter's ice, whose effects he observes at home, he passes to the heat of tropical, and the cold of arctic, regions, with the luxuriant vegetation of the one and the stunted growth of the other; the plants and animals of his own country, with their interesting habits, serve as a standard by which he may estimate their representatives in other countries; and the notions of the adaptations of labor, and the modes of life, which he forms from what he sees around him, are drawn upon to construct pictures of industry and the habits of other races of his fellowmen. Then it is largely by the imagination that a knowledge of life is gained, whether of individuals or of communities. The life of home or school, and the life of society so far as the child sees it, are limited in their incidents; yet the teacher hesitates not to tell or read the story of human life, in its spheres and with its diversified enjoyments, in the belief that the pupil's experience, narrow as it may be, will enable him to realize the emotions portrayed, and gather up the lessons suggested. Biography and history are the natural sources of supply for materials of this sort; narratives of adventure by sea or land; descriptions of manners and customs; incidents in the life of men or societies which embody the virtuous emotions of our nature. Ideal life may come in to increase the store of materials; it is equally rich in instruction with the life recorded in history itself."

60. The happiness of childhood depends almost wholly upon the use of the imagination. The plays of children are nearly all imitations of the real work of older people, and

serve as a preparation for the later actual duties of life. Thus they make dolls, dress them, feed them, put them to bed and rock them to sleep ; they cook, bake, set tables, wash dishes, clean house, make parties, receive callers, build dams, mills, houses, railroads, forts ; ride horses, keep store ; act soldiers, doctors, preachers, teachers, etc.

REMARK.—The statement is sometimes made that the imagination is more active in childhood than it is at any later period of life, but such an opinion rests upon superficial observation or investigation. The imagination is not stronger, or more active, in early life than it is at a later period ; on the contrary, it is weaker, but its activities are more noticeable, because nearly all of them are, of necessity, imitations of the actual work of grown people.

61. The use of the imagination is required in every calling in life. Nothing can be done intelligently and skilfully without a mental picture as a pattern. The mechanic, the dressmaker, the baker, the cook, the farmer, the doctor, the lawyer, the orator, the preacher, all must use it to meet with success in their vocations.

62. To feel with a person and for a person, it is necessary to imagine ourselves in his place. The chief reason why some persons seem to be unsympathetic is that their imaginative powers are sluggish or dull ; they cannot place themselves in the position of those who are in sorrow or distress.

63. The imagination is the faculty which constructs forms of beauty ; it is therefore one of the powers that give culture to taste and refinement.

64. James Freeman Clarke says : " No man can be wholly unhappy who is accustomed to look for beauty in nature and in human life. His is a joy which never wearies.

" All mere drudgery tends to stupefy the imagination ; and all work is drudgery which is done mechanically, with the hand and not with the mind ; when we are not trying to do our work as well as possible, but only as well as necessary.

Such work stupefies the ideal faculty, quenches the sense of beauty. 'No matter how lowly the labor may be, if a man performs it as well as he can, he is an artist.' But when a man tries to shirk his work, when he does it in a slovenly manner or way, not as well as he might, then he becomes a drudge, even though his work be that of a poet or a sculptor. He ceases to exercise his ideal faculty, and stupefies it. Then the sense of beauty dies out of his mind.

"If men are taught to look for beauty in all that they see, to embody it in all that they do, the imagination will be both active and healthy. Life will then be neither a drudge nor a dream, but will become full of God's life and love, and we will be brought into the love of that divine beauty which is above all, through all, and in us all."

65. Explanation of Terms.—1. Analysis consists in separating a complex whole, whether material or mental, into its elements—the parts of which it is composed. 2. Synthesis consists in forming a whole of its parts. 3. Abstraction is mentally withdrawing certain characteristics or qualities from objects. It may also be regarded as the withdrawing of the mind from all the other properties of objects except those under special consideration. 4. Generalizing is finding the general or common characteristics in a number of objects or concepts. 5. Classification is grouping similars into classes, and embraces generalization. 6. Comparison is simultaneously giving attention to several objects to discover their agreements or differences. 7. General concept is the term applied to a class, or to a group of all the common properties of the objects that compose the class. A general concept, therefore, cannot be imagined, it can only be thought. 8. To apprehend is to seize, to take possession of ; to comprehend is to understand. 9. A thing is known when it is assigned to its proper class. 10. Apperception is the appropriation of the new by the

old ; it is an organizing process. 11. Elaboration is another name for thinking.

66. Judging.—When the mind compares two objects or concepts directly with each other to determine their relation, the process is termed judging, and the result is a judgment. The objects compared may both be physical (material) or mental, or one may be physical and the other mental ; that is, we may compare one object (physical or mental) or act with another, or we may compare it with our concept of it or of what it should be. The concept or image is then taken as the standard, or measure, of the comparison. Determining whether a thing is sour, sweet, or bitter ; hard or soft ; cold or warm ; black or white ; weak or strong ; coarse or fine ; suitable or unsuitable ; good or bad ; right or wrong, are all acts of judgment.

“We judge whenever we affirm or deny one thing of another. Everything we know, or think we know, involves an element of judgment, and, when it becomes distinct knowledge, can be explicitly set forth in a proposition.

“An expressed judgment is a proposition.”—*Sully*.

67. Without the use of judgment, no advance step can be taken in the acquisition of knowledge. Judgment is among the earliest powers exercised by children ; every act of discrimination requires it—is an act of judging. Ideation employs it in completing its images.

REMARK 1.—Thinking is a general term applied to discovering relations ; and its three successive stages are conception, judging, and reasoning.

REMARK 2.—The several intellectual processes through which the material of knowledge passes from the concrete to the complete general concept are also usually given as three: 1. Comparison ; 2. Abstraction ; 3. Generalization.

REMARK 3.—The last step in the conceiving process is that of denomination, giving a name to the concept.

68. Judgments may be explicit or implicit ; that is, we may judge consciously or unconsciously.

69. Reasoning.—Not all objects or concepts can be directly compared with each other. Whenever direct comparison is impossible, a third object or concept must be found with which each of the others can be compared and their relation determined.

REMARK.—Frequently more than one intermediate term of comparison is necessary to determine the relation of the two objects or concepts under consideration.

70. When the relation of two objects or concepts is determined through one or more intermediate ones, the process is called reasoning. Examples of reasoning: 1. Given the cost of 4 lbs. of butter, to find the cost of 3 lbs. Here the intermediate term, or number, is 1 lb. 2. To find the number of men that can build 80 rods of wall in 16 days, if 12 men can build 50 rods in 15 days. Here there are two intermediate terms, rods and days; and the terms of comparison are one rod and one day. 3. A house and lot costing \$6750 were sold at a gain of $12\frac{1}{2}$ per cent.; how much was received for them? Here the term of comparison is 1 per cent. 4. On butter sold at 40 c. the gain was 25 per cent.; what was the cost? Here the term of comparison is 1 per cent. 5. Stealing is a violation of law; a violation of law is a crime; therefore stealing is a crime. Here the intermediate term is *a violation of law*. 6. Persons who tell falsehoods are not believed when they tell the truth; Alias tells falsehoods; therefore he is not believed when he tells the truth. Here the intermediate term is *falsehoods*.

71. Kinds of Reasoning.—There are two kinds or modes of reasoning, the inductive and the deductive. When a general truth is inferred or discovered from the examination and comparison of a number of individual facts the process is called induction; and when a particular truth is found through a general truth it is called deduction. The application of general principles to cases that come under them

or are included in them is also deduction. When the child finds that a hot stove, a gas-flame, a hot coal, etc., burn its fingers, and that fire is the general cause of the heat, it comes to the conclusion that fire burns; and it reaches this conclusion inductively. Afterwards when the child keeps its fingers from hot objects, it does so because it reasons deductively that all hot objects burn.

72. All the known laws that govern the material universe have been learned by induction. The naturalist cannot make nature's laws; he can only discover them.

73. Leading pupils to discover their own rules, principles, and definitions, from the examination of a number of special cases, is inductive teaching. The deductive method is the reverse of the inductive; by this method pupils are taught to apply rules, principles, and definitions.

REMARK.—The inductive is the method of acquiring knowledge; the deductive, of using it.

74. The Quantity of General Concepts.—Concepts, being aggregates, or syntheses, of attributes or properties, are said to have quantity. By the intensive quantity of a concept is meant the sum of the qualities which constitute it and distinguish it from other classes of concepts; and by the extensive quantity is meant the sum of the concepts that may be classed under it. Thus, a quadruped is an animal having four feet; four feet constitute the intensive quantity; and the number of animals having four feet, the extensive quantity of the term quadruped.

75. Formal Reasoning.—Every logical form of argument (syllogism) or reasoning consists of three propositions, judgments, or statements, called, respectively, the major premise, the minor premise, and the conclusion. The subject and the predicate of a proposition are called its terms; the major term being that of widest scope, or extent, in the major premise; the minor, that of least extent in the minor premise; and the middle, that found in both the others,

and through which they are compared. The middle term includes the minor and is itself included in the major. Example: A is B (is included in B). C is A (is included in A); therefore C is B (is included in B). Or, taking a concrete example: Man is mortal; Smith is a man; therefore Smith is mortal.

76. Systematization.—Arranging concepts or classes of objects according to some order or plan is called systematizing. Arranging the classes or the grades of a school, the goods in a store, our daily work, etc., are examples of systematizing.

77. Intuition.—Intuition means immediate beholding. We can immediately behold facts of matter and of mind, and we can immediately behold, or cognize, necessary and universal principles. The senses behold facts of matter and of mind; and therefore give us empirical intuitions, or percepts; the Reason beholds necessary and universal principles, and gives us rational intuitions.

78. Of the correctness of no other knowledge can we have less doubt than of our rational intuitions. For example, we can for a certainty know that things which are equal to the same thing are equal to each other; that the whole of anything is greater than any one of its parts; that every effect must have a cause; that no object can exist without time or space.

REMARK.—A principle or truth is necessary when we cannot think its contrary; it is universal when it is believed and true throughout the universe—when it has no exceptions.

79. Sources of Knowledge.—As has already been shown, the mind has three sources of knowledge, perception (empirical intuition), thought (logical conclusions), and the Reason (rational intuitions)

REMARK.—Perception, memory, imagination, comparison, abstraction, generalization, conception, judging, reasoning, systematizing and rational intuition, belong to the intellect, the knowing power,

CHAPTER II.—THE FEELINGS.

80. Numerous divisions of the feelings have been made by psychologists ; the following, from Dr. A. Schuyler's Empirical and Rational Psychology, are among the simplest.

1. Physical : sensations, instincts, and appetites. 2. Vital : feelings of rest, as fatigue, of vigor or languor, and of health or sickness. 3. Psychical : emotions, affections, and desires.

REMARK.—Sensations are feelings. A sense is an organ through which we feel ; hence the senses are feelers.

81. Our feelings may also broadly be divided into pleasures and pains, or into physical feelings and mental feelings. Physical feelings come from affections of the physical organism, and mental feelings from cognitions and thoughts.

Mental feelings, in general, without regard to the usual subdivisions, will alone be considered here.

82. Mental feelings are both causes and effects ; as causes they move the mind to action, and as effects (as before stated) they result from cognitions and thoughts.

83. Our pleasures and our pains, our joys and our sorrows, depend upon our feelings ; or, perhaps better, are themselves states of feeling. Whatever produces pleasant or agreeable feelings is done with comparative ease ; and whatever causes painful or unpleasant feelings is wearisome and exhaustive. Whatever excites the feelings pleasantly is said to be interesting ; and whatever is interesting strengthens us and hence aids us in the performance of duty. Discouragements, worry, or whatever else depresses the feelings, reduces strength and is a hindrance to the performance of work. Anticipation or hope of success

excites pleasurable feelings and consequently lightens labor. Expectant enjoyment as the fruit of success strengthens us and enables us to work with ease.

84. With children, the time that intervenes between the performance of duty and its resultant enjoyment must be short ; their experiences are too limited to expect much enjoyment from that which is far off and, to them, of doubtful realization.

85. That which affords immediate enjoyment is its own stimulus to interest and exertion ; but that which gives only anticipated, distant realization of success and enjoyment needs to be invested with novelty and interest to excite curiosity—a desire to know—as a condition to lively attention and exertion.

86. The motives that produce pleasurable emotions in some natures do not do so in others. The love of praise urges some to action ; hope of success, others. A student will study hard, day in and day out, early and late, to attain the end he has in view—prosperity or eminence. A man will perform disagreeable as well as severe physical labor if it will minister to his ultimate comfort or enjoyment. It is evident, therefore, that our feelings play an important part in the performance of all kinds of work,—especially in that of education—and that the motives which excite them and urge the student to action should be carefully studied by every one who desires to become an intelligent and a successful teacher. Garvey, in his *Human Culture*, says : “To interest pupils in their studies is the great secret of success in teaching ; but the interest of the pupils is best awakened by exciting their curiosity, by holding out to them, in a pleasant manner, whatever of strange, new, and wonderful the proposed study contains to reward their perseverance and attention. * * * The self-sustaining power of pleasurable feeling ought to be a grand lever in the hands of the educator. If we once make the

subject of study a source of pleasure, we may safely leave the mastery of it to the spontaneous energy of the pupil's mind. * * * By exciting the emotions we excite the inner forces of the mind, which cause it to expand and unfold its faculties just as the influence of the seasons excites the dormant forces of the plant."

REMARK.—Excitement, as here used, does not mean passion, but such interest or pleasurable feeling as shall carry the student through his work with comparative ease, such as shall arm him with strength and urge him onward to the reward of his labor.

87. Character.—What a man really is constitutes his character; what others think of him, his reputation. Character depends largely upon the feelings; and since character is the man—makes him either worthy or unworthy of the confidence and respect of his fellow-men—the moral feelings cannot be overlooked in a course of instruction.

88. The feelings cannot be educated; they are not thinking powers; they can only be trained, and they must be so trained that right conduct, right and worthy motives, shall afford pleasure; and wrong and unworthy motives and conduct, displeasure and pain.

89. The character of the teacher has much to do with the moulding of that of his pupils. He must be consistent; he must himself be and do what he would have his pupils be and do. He must not only hold up for their admiration that which is noble and good, but must exemplify it in his own daily life. He must show not only displeasure, but positive abhorrence, of that which is ignoble, wrong, mean, or debasing. Young people, especially children, are imitative beings, and both consciously and unconsciously acquire the habits of those with whom they are frequently associated. They are in the presence of the teacher five to six hours a day during not fewer than six months of the year, and, too, during the most pliable period of their lives; that his influence must be the main factor in shaping their

thoughts and habits must therefore be evident to all who properly consider the matter. Hence his character should constitute one of the principal elements of his qualification.

90. Good conduct comes from good thoughts, and good thoughts come from associations and surroundings that inspire them or prompt them. The mind is ever active ; it must have something to act upon. Those who have the charge of children must provide this material. If they fail to do so, the children will make their own selections ; and, owing to their inexperience—their inability to judge correctly as to what is best for them—and the natural tendencies to prefer present and temporary enjoyment, to future permanent good, they will, with the rarest exceptions, choose that which will in the end prove harmful—destructive of success and happiness.

91. Beautiful surroundings, such as well-arranged and well-kept school-grounds, neat and clean school-rooms, tastefully decorated walls, are important factors in the cultivation of good taste and right feelings. Nor must the teacher's own appearance be left out of the consideration. As has already been stated, but will bear repetition, children are imitative creatures, and both consciously and unconsciously adopt many of the habits and traits of those who are their daily associates. The teacher's influence may therefore justly be considered even greater than that of the parents ; for, besides being the instructor of the children, during their school days he is continually in their presence, and this causes them to be impressed more or less firmly not only with his appearance, but even with his modes of speech and action.

92. Children should be trained to conscientiousness ; they should be led to take pleasure in that which is right and good, and to hate that which is wrong and bad. Landon, in his *School Management*, says : " Although the germ of the conscience is born within us, it depends almost en-

tirely upon the kind and extent of the moral, religious, and intellectual training we undergo as to what strength and correctness of action it shall attain. We should therefore lose no opportunity of associating emotions of pleasure and satisfaction with right doing, of enlightening the children as much as possible as to the nature of various actions, of strengthening the judgment by suitable exercises, and of ennobling the sense of duty. The cultivation of the conscience so that it may be a sure guide to the children, and that they may readily obey its dictates, should engage the teacher's attention throughout the period of school life. This training is for the most part indirect, but it is none the less important and should be none the less sure."

93. The feelings cannot decide as to what is right or what is wrong ; decisions belong to the knowing powers. The feelings can only prompt the will to choose and to act in accordance with the decisions of the intellect. The feelings are moved by the intellect, and can be changed alone by it. Good thoughts produce good feelings ; bad thoughts, bad feelings ; sad thoughts, sad feelings ; pleasant thoughts, pleasant feelings ; angry thoughts, angry feelings. Angry feelings can be changed to good or pleasant feelings by changing the thoughts that give rise to them to others of a good or pleasing nature.

94. We are responsible for our thoughts and feelings. The fact that we can change them proves it.

95. Strong feeling, excitement, or passion prevents sound, sober thought. It is impossible to reason with an angry person. If we desire to reason with him, we must tell him an amusing anecdote or something else that will change the subject of his thoughts. The same (as before stated) applies to pupils in a school. If any of them are angry or sulky, they cannot study. The remedy is a change of thought, by means of an anecdote, an interesting talk or conversation, an illustration, or an experiment.

REMARK.—Conscience includes knowing and feeling.

CHAPTER III.—THE WILL.

96. The only power of the mind whose operations remain to be considered is the will. This is the power that directs, chooses, and acts; it is the executive power of the mind. It directs the intellect—gives it its work, sets it in motion, and waits for its conclusions. If the thing considered by the intellect is capable of ministering to our gratification or well-being, a desire or longing for it follows. It remains, then, for the will to be governed by this desire, or to refuse to do so, in view of other considerations.

REMARK.—One of the distinguishing characteristics of will is a *conscious* effort to attain a *known* end.

97. The manifestations of will may be classed under two heads, untrained and trained; in other words, brute and rational. An untrained will is one that is governed by impulse or passion. A person under the influence of such a will disregards the light of judgment and reason, and is therefore said to act contrary to reason, to be unreasonable, stubborn, obstinate, headstrong, ungovernable. This state of will is that of children and of the uncultured—an evidence of an untrained mind. A rational will is one that is not hasty, waits for the decisions of judgment and reason, and is governed by the highest good of the individual. Such a will is said to be strong when it inflexibly ministers to the best interests of the person, as dictated by the decisions of the intellect. It is weak when it permits itself to be controlled by the appetites or by any unworthy motives. The glutton, the tobacco smoker and chewer, the opium-eater, the inebriate, etc., have weak rational wills. Like children, they are under the domination of their ap-

petites, and the longer they permit themselves to be thus ruled the weaker the will becomes.

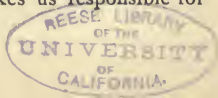
98. When our appetites and our passions gain the mastery over us, they become tyrants, dethrone the will, and reduce us to a state of imbecility—pitiable objects! No one need, however, be under the power of his appetite or controlled by any debasing motives. The will, like other powers, can be trained; but the training must begin early and continue as long as may be necessary for the individual to acquire self-control.

99. One of the first things a child should be taught is unquestionable obedience to those in authority over it. Besides obedience, neatness and cleanliness of person, truthfulness, self-respect, respect for the rights of others, politeness, and kindness should early be impressed upon its mind. Permitting a child to have its own way is no kindness to it. On the contrary, it is doing it a lasting harm. The will is trained by rigidly requiring the right to be done and the wrong to be shunned. In this way right-doing becomes a habit, and this habit is a right will.

100. Since the feelings urge the will, the importance of the formation of right feelings or desires becomes apparent; for if our desires are in harmony with our best judgment, and their force is of sufficient strength to move the will in the same direction, we have our powers under perfect discipline or control.

101. The will is simply the person willing or exercising the power of willing. He is at liberty to select and to do that which will minister to his well-being, either physical or spiritual, or that which will prove destructive of it. The will, therefore, is an important factor in the formation of character.

102. The power to will, to select one of several duties, acts, or things, implies the ability or freedom to reject any or all of them, and consequently makes us responsible for



our acts. Responsibility does not, however, apply only to overt acts, but even to thoughts. This is evident from the fact that the will can change the subject of thought, or can direct what it shall be.

103. Since good conduct comes from good thoughts and bad conduct from bad thoughts, the importance of having the mind occupied with wholesome thoughts is too plain to require further proof.

REMARK.—Here, again, the importance of beautiful surroundings, suitable associates, and wholesome literature or reading-matter becomes evident.

104. Bad thoughts can be expelled from the mind only by the introduction of good ones—the good will drive out the bad, and similarly the bad will drive out the good. Here, again, we see that we are responsible for our thoughts.

105. As before stated, the will and the feelings depend upon the intellect. The feelings are excited by the thoughts, and the will is moved by the feelings. The feelings of all persons are not, however, excited with equal ease. Some are moved very quickly, others very slowly, and between these extremes many grades are found. Those who are easily or quickly excited are said to be passionate, impulsive; they act without proper consideration, without waiting for a complete report or decision from the intellect. Rash or impulsive persons are neither safe counsellors nor safe guides; for they frequently do things of which they afterwards are ashamed, and for which they are sorry.

106. In addition to the foregoing statements concerning the importance of training the will to depend upon the intellect for its course of action, it should be emphasized that this dependence must be formed into a habit.

107. HABIT.—Habits control nearly everything we do, and it is only after we can do a thing from habit that we can do it with ease and pleasure. The use of good language, all mechanical executions, thinking good thoughts,

controlling our feelings, and acting from pure motives must largely become matters of habit. Teachers must invariably insist upon such conduct from their pupils as is reasonable and right ; and this course must be continued until right-doing has grown into a habit—until the pupils can be left to govern themselves.

108. However necessary outward means of government may be during the early periods of life, the fact must not be overlooked that they effect no radical change in the disposition : they serve simply as a restraint. A change of disposition must come from within, and must give stability to habits of doing right. Hence the further pupils advance in intellectual and moral culture—in their ability to take care of themselves—the less care need be exercised over them by parents and teachers.

109. It cannot be too deeply impressed upon the minds of teachers that the best book upon psychology is the living, acting child. Child study, or experimental psychology, is bearing fruit for the teacher's guidance that speculative psychology never dreamed of.

110. Teachers who desire to read more to aid them in their studies than is contained in the foregoing notes would do well to consult Dr. W. O. Krohn's "Practical Lessons in Psychology" and Prof. William James' "Briefer Course in Psychology." Those who wish to acquaint themselves with the Herbartian psychology and pedagogy should read Lange's "Apperception" (edited by Dr. Charles De Garmo), Dr. Charles A. McMurry's "General Method," or Rein's "Outlines of Pedagogics."

Part III.

IMPORTANT OBSERVATIONS AND INFERENCES.

1. Education begins and ends with life.
2. The object of scholastic education is to teach the pupil how to learn. Its methods should therefore be such as will enable him to carry on his own education pleasantly and successfully.
3. Education cannot create powers or faculties : it can only develop those that exist.
4. The mind can develop what is in itself only by its own activity ; self-activity (exercise), therefore, educates.
5. The powers of the child can be exercised by no one but the child itself ; consequently, not what the teacher does for the child, but what it does itself, educates it.
6. The inner promptings of the child make themselves known by outer manifestations.
7. The child indicates its own mode of receiving instruction : hence the teacher learns from the child how to teach it.
8. It is natural for a child to seek knowledge.
9. Gratifying a child's desire for knowledge stimulates that desire.
10. Knowledge begins with experience ; the concrete should therefore precede the abstract—things should precede their signs or names.
11. Facts and phenomena should come before laws and principles.
12. Thought should come before expression.
13. Each mind has its own rate of growth and development.

14. Substantial learning or attainments cannot proceed faster than the mind's rate of growth and development.

15. Attempting to go too fast, or going too slowly, weakens the mind.

16. There is an intimate relation between the body and the mind ; work of either reduces the power of the other.

REMARK.—Bain, in "Mind and Body," says : "The fact is now generally admitted that thought exhausts the nervous substance as surely as walking exhausts the muscles."

17. Rapidly growing children tire easily ; their rapid growth reduces their power of endurance.

18. "The time to acquire skill in the use of any power, mental or physical, is when the power is growing."

19. "The time to teach a thing is indicated by the arrival of the child's interest in it. Children's interests change with age. What interests them at an early period does not do so at a later."

20. Attention, memory, and imagination are not so many separate entities, but rather necessary accompaniments or aids to the other powers. We may have as many memories and imaginations as there are classes of things to remember and to image, each requiring its own peculiar training.

21. Before a teacher charges his pupils with inattention, he should ascertain the predominating kind of mental images they form, whether visual or auditory.

22. Poor remembering means in most instances poor images and understanding, for which teachers should largely hold themselves responsible.

23. Before the teacher begins to instruct, he should ascertain the contents of his pupils' minds, or he may build without a foundation.

24. The present must grow out of and upon the past. What the child has not experienced it cannot image or comprehend.

25. Every percept is made of present and past experi-

ences. All our mental activities are performed with accumulated capital ; that is, every thought we think receives the benefit of all our past thinking. That is apperception.

26. Instruction aims at power and skill ; education at character.

27. Adaptation of the subjects of instruction to the pupils' growing needs is the key to success in teaching.

28. How a subject is taught is more important than what is taught.

29. The teacher's life should be an example for his pupils, and his success should be measured by his moral influence.

30. The end of school government should be self-control and character.

Herbartianism.—The five methodical steps which, according to Herbart and his disciples, must be taken in teaching a lesson : 1. "The preparation [analysis] ; 2. The presentation [synthesis] ; 3. The combination [association] ; 4. The recapitulation [system] ; 5. The application."—*Lange's Apperception.*

Part IV.

OBJECT-LESSONS.

A. THEIR DESIGN.

- a.* The training of the senses—observation.
- b.* The gaining of knowledge.
- c.* The development of the power of thought.
- d.* The cultivation of expression.

B. THE PLAN OR METHOD OF A LESSON.

- a.* Have a well-defined end in view.
- b.* Select a suitable object to be used.
- c.* Determine the method or order to be pursued.
- d.* Lead the pupils to make their own discoveries.

REMARK.—There is no better way of enlarging the children's vocabulary and of increasing their stock of knowledge than by means of lessons on objects (things). With such lessons they can be taught the names, properties, relations, and parts of objects. By "object-lessons," however, is meant lessons with objects, not simply about them. The children themselves must examine the objects. Knowledge acquired in this way is more certain and permanent than that which is obtained at second-hand—from books or from teachers.

Children could profitably spend months upon lessons of this kind before receiving instruction of any other character.

Part D.

PENMANSHIP.

REMARK.—The doing of anything is best learned by intelligently directed practice, and this applies to nothing more forcibly than to penmanship. Every one, unless paralyzed or deformed, can learn to write a neat hand. The wretched writing found in most schools and among many persons otherwise claiming to be educated is an unmistakable sign of defective teaching.

1. **Suggestions.**—Proper position of the body, correct penholding, and the best work the pupils are capable of doing, *must be insisted upon* ; and all their writing, until it is as nearly perfect as they can make it, should be considered practice in penmanship.

2. Every letter should invariably have the same form, that of the so-called standard letters.

3. **Penholding.**—The penholder should be placed between the thumb and the first two fingers, so that it may rest before the third joint of the forefinger.

REMARK.—The penholders and pencils for children should, if possible, be no more than an eighth of an inch in thickness.

4. The thumb and the fingers should be bent outward so as to bring the end of the thumb opposite the first joint of the middle finger.

5. To keep the pen at the proper angle, the thumb should be pressed a little under the holder.

6. The left side of the middle finger should support the holder just above the pen, and the forefinger close over the holder.

7. The pen should be held as lightly as possible.

REMARK.—The suggestions on penholding apply equally to the holding of pencils.

8. **Position of the Body.**—The body should be erect—not resting on the arm that carries the pen—the head slightly inclined forward to see the writing.

9. The forearm should rest on the muscle in front of the elbow and the hand on the nails of the third and fourth fingers. The wrist should not rest on the desk or the paper.

10. If the hand is so held that the wrist is horizontal, both sides at equal distances from the desk, the penholder will point to the right shoulder and give the right slant to the writing.

11. To enable the children, in their early efforts, to write in straight lines, their tablets should be ruled with base or guide lines.

12. Penmanship should be commenced with copying the first lessons in reading, and that a good beginning may be made, the teacher should show the children how to hold the pencils (or the crayon, if they write upon the blackboard), and, when necessary, how to form the letters, sometimes guiding their hands.

13. **Height of Letters and Spacing.**—The proportionate height of the letters, and proper spacing of letters and words, should not be overlooked.

14. **Flourishing.**—Until the pupils can write a neat, plain hand, no efforts at flourishing should be tolerated.

15. **Charts.**—Penmanship charts should be placed upon the wall above the teacher's blackboard, where the pupils can at all times see the correct forms of the letters.

REMARK.—If the foregoing suggestions on penmanship be strictly carried out, neither copy-book nor classes in writing will be necessary, and better penmanship, in less than half the present usual time, will be the result.

Part VI.

PRIMARY READING.

1. **INTRODUCTORY REMARKS.**—Before an effort is made to give formal instruction to children, either in reading or any branch of knowledge, their confidence and good will must be gained. This may be done by engaging them in conversation upon something that is familiar to them and in which they take an interest. These conversations should be continued until the children's timidity has been overcome and they freely converse with the teacher. During these familiar talks as much knowledge should be drawn from them as possible. In this way, too, the teacher can learn the extent of their stock of knowledge, and this information will serve him as a basis for the superstructure of knowledge which, under his guidance and superintendence, they are to rear.

2. Reading is thinking—not mere word-calling; consequently the more thoughtful and intelligent the pupils are made by such preparatory lessons as those on objects, the better they will be prepared for reading.

3. Reading may properly be considered under two heads, impression and expression, or silent reading and audible reading. Silent reading has for its object the getting of thought, and audible reading that of conveying it to others. The getting of thought is the first and chief thing to be aimed at. When this end has been attained,—the thought found,—conveying it to others by means of audible reading becomes comparatively easy; for emphasis, pause, and inflection take care of themselves—the thought controls them.

4. Various methods of teaching beginners to read have from time to time been advocated and practised. Among them may be named the alphabetic, word, phonic, and sentence. But as no one alone of these methods is free from objections, it is found best to form a method that combines what is best in all of them.

5. The word method seems to be the simplest, and therefore the best to begin with. This method follows that of nature, presenting wholes before parts. It is also philosophical, for, in order to read, to get the thought, words must be recognized as wholes, each as a single picture. If either the names or the sounds of the letters, instead of the whole words, attract the attention of the pupils, they lose the thought, and merely pronounce the words.

REMARK.—Since, in order to read, children must recognize words, it is not only reasonable, but a saving of time, to begin with words.

6. Words with which the children are familiar should at first be taught. The words which they recognize through the ear they must now learn to recognize through the eye.

REMARK.—The length of time required to impress a word permanently upon the mind and to make its name of ready recollection depends upon the interest with which the teacher presents it. To make this work a success he must have a number of devices at ready command.

Suggestions for Teaching Primary Reading.—1. Select a word that is the name of an object in which the children take an interest. Engage them in a conversation about the object, to interest them, to gain their attention.

2. Write the word upon the blackboard. Tell them what you have written (its name) and urge them to observe it carefully, so as to be able to recognize it when they see it again.

3. Erase it and write it among others upon the blackboard. Tell them to find it.

4. By means of a variety of exercises of this kind and of others, impress the word upon their minds, always associating the form (written word) with its name.

5. Test the impression with word-cards, charts, and any other means which you may have at your command or which you may be able to devise.

REMARK.—Writing the word among others upon the blackboard affords a good test of the impression.

6. To make the impression permanent, the children should be required or urged to copy their lessons both upon the blackboard and into their tablets. The tablets used for this purpose should be ruled, and the lines should be not less than three eighths of an inch apart. As before stated, the pencils should be very thin—one eighth of an inch in thickness—to enable the children to hold them properly.

7. Every lesson should begin with a review of the preceding lesson or lessons. Every word taught should be reviewed from day to day until it is permanently impressed upon the mind and can instantly be recalled. The recognition of words should be made automatic.

8. After the first word has been taught, others should be taught to combine with it, or should directly be combined with it, to form a phrase or sentence.

9. As many new words should be taught at each lesson as the pupils can well learn.

10. The teacher should keep a memorandum of everything given to the class, to be used as material for reviews. He should also prepare as many slips of paper with the words which the children have learned written upon them, as he has pupils. These slips may be kept in pasteboard boxes, and whenever the children are at leisure—when they have performed their other tasks, or their assigned work—they should take out the slips and see how many of the words they remember.

11. The sentences written upon the blackboard as read-

ing exercises may also be copied upon slips of paper and kept in boxes for the children to read after their other work has been performed.

12. Lists of the words taught may be written upon the blackboard, where the children can, from their seats, see them. Of these they should be encouraged to make as many sentences as they can. The sentences should be brought to the class and kindly criticised—errors pointed out and improvements suggested. These exercises train children to the correct use of language.

13. As before stated, the words of the reading-lessons should at first all be taken from the children's vocabulary, or stock of words in use.

14. The statements, or sentences, should as far as possible be about something that interests the children. Statements about themselves frequently prove interesting to them.

15. The teacher should have a good supply of toy objects. At the recitation every member of the class should be given one, and the children encouraged or requested to talk about them. Their talks, or sentences, may be written upon the blackboard and read, or only the most suitable ones may be written upon the blackboard and read.

16. Number-lessons may also be made reading-lessons. The statements of facts which the children make may be written upon the blackboard and read, or they may write them, at their seats, into their tablets, bring them to class, and read them.

17. Lessons on size, form, weight, position, color, quality, etc., may be used in the same manner and for the same purpose as those on number.

18. The reading exercises should be varied from day to day; that is, of the same words as many different sentences should be made as possible. Varying the exercises not only lends interest to the work, but helps to impress the words upon the children's minds.

19. The sentences should at first all be short. Long sentences are too difficult for beginners ; their length prevents the children from readily grasping the contained thought.

20. The reading exercises, the words, phrases, and sometimes sentences, may be taken from a suitable book, a Primer or First Reader.

21. The pupils should at first read only from the black-board ; afterwards also their own written work, from their tablets, papers, or slates.

22. After the pupils have learned from one hundred to one hundred and fifty words by sight, by the word method, they should be taught the analysis of words into their sounds. They should be shown that the names of all words are made of combinations of sounds. This may be done by taking suitable words and pronouncing them slower and slower each succeeding time until the separate sounds are heard. Words in which all the sounds may be prolonged should at first be used.

REMARK.—Some teachers introduce the sounds of the letters and their signs almost from the first lesson in reading, and do it successfully.

23. The analysis of words into their sounds should be followed by the synthesis of the sounds. The children should be made acquainted with the letters that stand for the sounds, and should have practice in determining the pronunciation of words.

NOTE.—Teachers who desire a carefully worked-out system of primary reading, in general accord with the foregoing, will find "The Rational Method," published by Silver, Burdett & Co., New York, the most recent and the best. It is a thoughtful method, and the shortest to the mastery of words and intelligent reading.

24. All words that cannot well be taught by sounds should be taught by sight, or as wholes.

25. The English language contains about forty-three

different sounds, and has only twenty-six letters by which to represent them ; some letters, therefore, stand for more than one sound. The particular sound for which a letter stands is indicated by a sign placed upon the letter, above it or below it. These signs are called diacritical marks.

26. Whenever a letter is taught as the representative of a sound, its mark should be taught in connection with it.

REMARK.—The diacritical marks enable the children to discover the pronunciation of words.

27. Whenever a new word is introduced, its sounds should be indicated by their proper marks. In the dictionaries the pronunciation of words is indicated by diacritical marks. In some of the first reading-books diacritical marks are also used.

Sounds of the vowels and consonants as given by Webster :

VOWELS.

ā as in āpe	ō as in nōte
ă " " ăt	ō " " nōt
ā " " ārm	ō " " ōr
ā " " āsk	ô " " lôve
â " " cāre	o " " mōve
ā " " āll	o " " wōlf
ă " " whăt	o " " obey
ā " " senāte	ōō " " mōōr
ē " " mē	ōō " " bōōk
ē " " mēt	ū " " mūte
ē " " thēre	ū " " ūp
ē " " tērm	u " " fūll
ē " " they	u " " rŭle
e " " event	û " " bŭrn
ī " " īce	ÿ " " any
ī " " īt	ÿ " " hÿmn
ī " " fīrm	ÿ " " mÿrrh
ī " " pīque	

CONSONANTS.

c as in eat	ŋ as in finger
ç " " çent	ſ " " is
ch " " chorus	th " " these
g " " germ	x " " exist
g " " go	

28. As a further aid to pupils in discovering the pronunciation of words, silent letters may be marked, also double vowels and double consonants.

29. The names of the letters of the alphabet afford no aid to the children in discovering the pronunciation of words; they serve merely to designate the letters when speaking of them, and should therefore be taught only indirectly or incidentally.

REMARK.—Oral spelling cannot be substituted for real spelling, and if practised at all, should be postponed until it will not interfere with reading.

30. To familiarize the pupils with the forms of words and to afford practice in forming them and in penmanship, they should be required to copy carefully and neatly, into their tablets, either a part or the whole of every reading-lesson until they have completed the Second Reader; and the tablets or books should be preserved to note the progress of the pupils in penmanship, spelling, etc.

31. All new words and words not well known should be written upon the blackboard, and the pupils drilled upon them until they can readily name them. They should not be permitted to attempt the reading of a sentence until they can, upon sight, pronounce every word in it.

32. Instead of being asked to read a sentence or paragraph, they should be requested to tell what it says. They should generally tell it in their own language.

33. Conversational tones should be insisted upon in reading.

34. If a lesson contains anything worth remembering, the pupils should (with some exceptions) be required to give the substance of it in their own words. An exercise of this kind cultivates their memory, imagination, and the power of expression.

REMARK.—Sometimes, too, they may be required to substitute synonymous words and expressions for those found in their lessons.

35. Words and sentences may be dictated to the pupils to be written into their tablets or upon the blackboard, to test their orthography and to impress the correct word-forms upon their minds.

36. Owing to the similarity of the forms of letters, the change from script to print is not difficult. To make the transition, the teacher should place exercises in both forms of letters upon the blackboard and let the children compare them. A few days' comparison and practice in reading print will remove all difficulties in reading the latter.

37. As soon as the children can, with considerable ease, read from the blackboard, they should be allowed to read either from slips or cards printed for the purpose or from books; and until they can read fluently from either, they should have only one or two pages of reading-matter given them at a time. If they read from books, instead of slips or cards, they should be given them only during the recitation.

REMARK.—If the foregoing suggestions with reference to reading from slips or cards and books be carefully observed, the children will every two or three days receive something new to read, and the expectation of this will keep up their interest in reading.

38. The sentences should at first be commenced with small letters, and each letter should invariably have the same form—that of the so-called standard letters.

REMARK.—The teacher cannot be too careful with his own penmanship. As it is to be an example, a guide, to his pupils, it should as nearly as possible be perfect.

39. As soon as it is believed not to confuse the pupils, the capital letters should be introduced to begin sentences.

40. The pupils may read three, four, or more months only from the blackboard and their tablets or papers.

41. Only one thing should be introduced at a time, and the teacher should be sure that it is both understood and remembered before he introduces anything else.

42. Nothing less than the best work the pupils are capable of preparing should be accepted from them. Accepting carelessly prepared work tends to the formation of careless habits.

43. All the teacher's work should be an example for his pupils.

44. Articulation needs careful attention in every exercise in which the voice is used.

45. After the pupils are far enough advanced to do so, they may be required to find all the possible meanings of which a sentence permits by emphasizing in succession the different words in it.

46. The children should as early as possible read for information, so that a desire for reading may be formed. Much of the reading done in the majority of schools adds nothing to the pupils' stock of knowledge, neither possesses any interest for them, and consequently, in a correct pedagogic sense, does more harm than good.

Part VII.

ADVANCED READING.

REMARK 1.—Primary reading, as the term is used in these Notes, refers to that done on the blackboard and in the "First Reader."

REMARK 2.—If reading is well taught in the primary grade or class, that of the higher will largely take care of itself.

1. The reading-matter for children should, as far as possible, be of a thought-stimulating, interest-creating character, adapted to their comprehension and power of appreciation. Much that is now read in the schools falls far short of meeting these conditions.

2. As has already been stated, but will bear repetition, a child should not be permitted to read a sentence audibly until it has read it silently, and has obtained the thought. In other words, it should not be allowed to express the thought until it has it. Permitting, or, as is too frequently done, requiring, pupils to read, to express thoughts audibly before they have any—thoughtless reading—is chiefly responsible for the poor reading that is so general in all grades of schools.

3. Thought-getting and thought-expressing are not the same. A pupil may make commendable progress in obtaining thought, but be slow in expressing it. Words are slow in impressing themselves upon some minds. There are children, too, who learn more readily through the ear (ear-minded) than through the eye; they remember better what they hear than what they see. Instead, therefore, of chiding or scolding them for slowness, the teacher should ascertain its cause and, as far as possible, provide a remedy.

4. When pupils are far enough advanced to do so, they should, frequently at least, if not generally, be required to read a paragraph or selection silently, and then give the thought in their own words, orally or in writing. This method of teaching reading cultivates both thought and expression, and consequently has a greater culture value than reading as it is usually taught.

REMARK.—Good or expressive reading should be insisted upon in every class or subject in which a pupil is required to read.

5. **Pronunciation.**—(a) Careless or slipshod pronunciation seems to be the rule rather than the exception in the majority of schools. It is not an uncommon thing to hear such pronunciations as the following: *Fir* for *for*, *ben* for *been*, *ûf* for *of*, *sence* for *since*, *sepret* for *separate*, *evry* for *every*, *rhetric* for *rhetoric*, *generly* for *generally*, *easly* for *easily*, *practus* for *practice*, *cälm* for *calm*, *kin* for *can*, *blessid* for *blessed*, *goen* for *going*, *govner* for *governor*, *histry* for *history*, *cäve* for *carve*, *staum* for *storm*, *articlation* for *articulation*, *peticular* for *particular*, *prinsple* for *principle*, *artic* for *arctic*, *ax* for *acts*, *plitital* for *political*, *an* for *and*, *däg* for *dog*, *fourt* for *fourth*, *you-rî-zon-nears* for *your eyes and ears*, *hî-zov-ry-zup* for *his hour is up*, *tay-cary* for *take care*, *gavim* for *gave him*, *ovis* for *of his*.

(b) When errors like the foregoing have been hardened into habits they are difficult to eradicate; but they can be corrected by persistent daily practice on the vowels and consonants, first separately and afterwards in combination. A part of every instruction period, as long as necessary, should be devoted to phonic drills.

(c) A reader or speaker can make himself more easily understood by a correct and clear enunciation of his vowels and a distinct articulation of his consonants than either by a high pitch or by loudness.

6. **Emphasis.**—(a) Any method of giving special sig-

nificance to an expression is emphasis, and emphasis is the life of reading. Correct emphasis helps the hearer to the meaning of what is read, and incorrect conceals it, or hinders him from getting it. A knowledge of the leading principles of emphasis is therefore a necessity to good reading.

(b) Alexander Melville Bell, one of the highest authorities on expressive reading and speaking, says: "The laws of emphasis form a study of the highest intellectual value, which has been too little investigated and systematized. No other department of elocution can compare with this in importance ; yet not only has it been superseded in books by unnecessary rules for inflection, and in schools by thoughtless imitation, but these rules, and all exercises founded on them, constantly violate the laws of (rhythmic) accent. Here is one point in which almost absolute uniformity must prevail among all good readers. Set practice right in respect to emphasis, and inflection cannot go far wrong."

(c) The sense of what is read determines the emphasis to the reader, and the emphasis conveys it to the hearer.

(d) Misplaced emphasis perverts the meaning. Take the following, for example, and read it as here indicated by the italics, and as it is generally read, and note the sense: "In the beginning was the *Word*, and the Word was with *God*, and the Word *was* God." This rendering would lead us to believe that nothing existed in the beginning but the Word, and that at that early period, at least, it was God, but that at a later day it was changed to something else.

Read the foregoing as here indicated, and observe the difference: "In the *beginning* was the Word, and the Word was with *God*, and the *Word* was God."

(e) The fifth verse of the first Psalm is frequently read : "The *ungodly* are *not* so ; but are like the *chaff* which the *wind* driveth away."

Without saying anything about the accent on *godly* in *ungodly*, this reading permits of at least two meanings: 1. That there are two kinds of chaff—one driven by the wind, which the ungodly resemble; the other not so driven, which we may infer the godly are like. 2. That one kind of chaff is driven by the wind, the other by something else.

(f) In *Acts* 20: 16 is found a good example to illustrate the variety of meanings of which a sentence permits by changing the place of emphasis.

1. *Paul* had determined to sail by Ephesus.
2. Paul *had* determined to sail by Ephesus.
3. Paul had *determined* to sail by Ephesus.
4. Paul had determined to *sail* by Ephesus.
5. Paul had determined to sail *by* Ephesus.
6. Paul had determined to sail by *Ephesus*.

The first sentence states *Paul's* determination; the second, what it *had* been; the third, that his mind had been fully *set*; the fourth, that he meant to go by *water*; the sixth, the *route*—by way of *Ephesus*.

REMARK.—Requiring children, or even more advanced pupils, to read all the various meanings of which a sentence permits by shifting the place of emphasis, is one of the best ways of breaking up monotonous reading.

For this purpose a sentence like the following may sometimes be used in connection with their regular reading exercises: Did you see Henry this morning?

(g) The following are given in some reading-books as examples of the use of the circumflex, but the efforts that are usually made to read them are in few instances successful. Properly applied emphasis, however, removes the difficulty, sets the inflections right, and expresses the intended sense.

A man who is in the daily use of ardent spirits, if he does *not* become a drunkard, is in danger of losing his health and character.

Stress upon *not* makes good sense; but if placed upon drunkard it would make inebriety a necessity to the preservation of health and character.

The dog would have died if they had *not* cut off his head.

Stress upon *not* expresses the sense, but if placed upon both *died* and *head*, we would be told that cutting off his head saved his life.

(h) The kind of emphasis depends upon the kind of thought to be expressed.

7. **Pausing.**—(a) Pauses, or cessations of the voice, are required in reading to enable the hearer the better to grasp the meaning of what is read.

(b) The pauses are governed by the sense and not by the punctuation-marks; hence, though cessations generally occur at these marks, they are also required where such points would be out of place.

(c) In the examples which follow pauses are indicated by the dash.

(d) Now—about that time—Herod—the king—stretched forth his hands—to vex—certain of the church. (*Acts* 12 : 1.)

The omission of the pause after *Herod* would signify that one of several persons by that name is meant.

The pause after *to vex* might be omitted, but its use adds force to the expression.

(e) But—the Jews which believed not,—moved with envy,—took unto them certain lewd fellows—of the baser sort. (*Acts* 17 : 5.)

Read without a pause after *Jews*, conveys the sense that those of the Jews who believed not were moved with envy; and with the pause, that none of them believed. A pause might also be made after *them*, but its omission does not add uncertainty to the meaning.

(f) There was a man sent from God,—whose name was John. (*John* 1 : 6.)

The omission of the pause after *God* might convey the meaning that the man was sent from the God whose name was John.

(g) And—turning the cities of Sodom and Gomorrhah into ashes—condemned them. (2 *Peter* 2 : 6.)

And, the first word, belongs to *condemned* (in reading), not to *turning*, and this the pauses serve to show.

8. Grouping.—(a) Words are spoken either singly or in groups. Those that express a unit of thought must, by the voice, be grouped into a unit of expression, and read throughout, without interruption, in the same tone as if they were one word.

(b) In the following examples the grouped words are placed within brackets:

(c) Forasmuch then as the children are [partakers of flesh and blood], he also himself likewise [took part of the same]; that through death he might destroy [him that had the power of death], that is, the devil. (*Hebrews* 2 : 14.)

Each of the expressions within brackets may be regarded as equivalent to a word.

(a) I charge thee therefore before God, and the Lord Jesus Christ, [who shall judge the quick and the dead at his coming]; preach the word. (2 *Timothy* 4 : 1, 2.)

The clause within brackets is subordinate to what follows (preach the word), and this must be made manifest in reading, by grouping and tone of voice.

(c) [Men shall buy fields for money, and subscribe evidences, and seal them, and take witnesses in the land of Benjamin, and in the places about Jerusalem, and in the cities of Judah, and in the cities of the mountains, and in the cities of the valley, and in the cities of the south]: for I will cause their captivity to return, saith the Lord. (*Jeremiah* 32 : 44.)

The substance of all within the brackets is, that prosperity shall again return to the land.

9. Etymology.—Etymology should receive attention in connection with reading.

REMARK 1.—A good reader reads thoughts, not words. He pictures to himself his author's views and expresses them as if they were his own. He is unconscious of everything but the thought. If pronunciation, inflection, holding of book, position of feet or hands, or anything else, diverts his attention, he loses the thought and reads like a machine.

REMARK 2.—Reading cannot be taught by rules; it requires a competent teacher, one who can himself read and teach, and who can discriminate between the useful and the useless.

Teachers who desire to become effective readers, but cannot avail themselves of the services of a competent instructor, will find Alexander Melville Bell's "Principles of Elocution" an excellent work for self-help.

Part VIII.

NOTES AND SUGGESTIONS ON TEACHING THE ENGLISH LANGUAGE.

CHAPTER I. GENERAL CONSIDERATIONS.

1. Like all other languages, that of the English-speaking people conforms to laws, and these laws are laid down, or taught, in books on English grammar.

2. The laws, or principles, of grammar are inductions from a study of the mechanism of the language ; and, like those of the material universe, are not made, but discovered.

3. The laws of language, as found in books on grammar, rhetoric, and logic, may be pursued by themselves as sciences, or they may be introduced as they are needed to enable the student to acquire a correct, an intelligent, and a skilful use of the language.

4. Prof. W. D. Whitney, in his *Essentials of English Grammar*, says : " It should be a pervading element in the whole home and school training of the young, to make them use their own tongue with accuracy and force ; and along with any special drill directed to this end, some of the rudimentary distinctions and rules of grammar are conveniently taught. But that is not the study of grammar, and it will not bear the intrusion of much formal grammar without being spoiled for its own ends. It is constant use and practice, under never-failing watch and correction, that

makes good writers and speakers ; the application of direct authority is the most efficient corrective. Grammar has its part to contribute, but rather in the higher than in the lower stages of the work. One must be a somewhat reflective user of the language to amend even here and there a point by grammatical reasons ; and no one ever changed from a bad speaker to a good one by applying the rules of grammar to what he said."

5. From the foregoing it seems clear that a distinction must be made between acquiring the skilful use of a language and the study of its mechanism. The former comes from practice, under careful supervision, and is an art ; the latter is the result of study, and is a science. The science and the art of language are therefore different things, and are pursued for different ends.

6. The advocates of technical grammar as a general school study but a few years ago claimed that it teaches its learners to speak and to write the English language correctly. When, however, it was asserted by those who had given the subject earnest consideration, that it failed to prove its claims, its advocates maintained that its shortcomings must be attributed to the teaching, and not to the subject. But this ground having, both by experience as well as by a careful study of the subject, been found untenable, has now been abandoned by the foremost thinkers and students of education.

7. Since, therefore, it has been found that the study of English grammar does not, in fact cannot, make good writers and speakers, much less importance is placed upon it than formerly.

8. Under the topic of Grammar in City Schools, Vol. I of the Report of 1888-89, of Dr. W. T. Harris, Commissioner of Education, has the following in agreement and confirmation of the foregoing, viz.: "In the admitted fact that the formation of habits of correct speech is not de-

pendent upon a knowledge of the rules and definitions relative to the construction of language and their mutual dependence, is found the justification for the lessened weight attached to such study."

The Report quotes the following from Fitch's Lectures on Teaching: "The practical art of using language in speech or writing with good taste and correctness . . . is probably best to be attained by talking to the pupil, by taking care that he hears little but good English, by correcting him when he is wrong, by practising him much in writing, and when he makes a mistake, by requiring him to write the sentence without one. It will certainly not be attained by setting him to learn Murray's, or indeed any other grammar."

The article on the study of grammar closes with the following important statement, viz.: "The art is the thing directly useful: the science has no obvious relation to practical affairs. The ability to speak and write correctly is not only desirable, but essential in every walk of life; the technical rules of etymology and syntax are almost valueless in themselves. Therefore, in accordance with the movement whose progress is here recorded, the art, that is, the practical, increases in importance in the course of study, while the science, that is, the disciplinary, decreases in the same proportion."

9. It is true that formal or technical grammar has some disciplinary value, but the amount of it is usually greatly overstated, and unfortunately so, for much of this supposed value is responsible for the sad condition in which the study, if it may be so called, leaves the minds of the children who, reluctantly, spend their time upon it.

10. Instead of dividing the study of language into grammar, rhetoric, and logic, and pursuing each as a separate subject, all should be taught under the head of *language*, and each at the proper time made to contribute its share to

the perfection of the whole, namely, the correct, elegant, and forcible use of it in writing and speaking.

REMARK.—The last paragraph refers to the work of schools below the college.

11. The use of good language must become a habit—"an unconscious one." But habits are of slow formation; that of language requires years. A correct taste, the ability to form a correct judgment of a composition, must be acquired. Like misspelled words, badly constructed language must offend the taste; that is, its defects or faults must at once appear on being seen or heard. No one, therefore, whose taste is uncultivated can teach language. He may be able to teach grammar, and to some extent rhetoric and logic; but he cannot tell whether discourse, of whatever kind, is well constructed, and therefore can be of no service to learners in the formation of taste, or in acquiring the use of good language.

12. From the foregoing considerations the conclusion seems evident that language should be taught by practice, intelligent practice, upon the principle that we learn to do a thing by doing it, and not by merely learning about doing it.

REMARK.—As before stated, all the technical grammar necessary to a correct, forcible, and intelligent use of English not only can be acquired in connection with reading and composition, but should be so acquired.

CHAPTER II.

I. ORAL LANGUAGE.

REMARK.—Spelling, penmanship, punctuation, and capitalization belong to composition, and should be taught in connection with it, and not as separate branches of study.

1. In order that children may talk, they must have something to talk about ; that is, they must have a subject for conversation.

2. The subject should be one that will interest them and that will add to their stock of knowledge. Stories that add nothing to their knowledge, that do not improve their powers of observation and thought, or contain anything worth remembering, should generally be avoided.

3. The subject should be one that the children understand, or can readily be led to understand. It should generally be one which they have observed or can observe. It should be thoroughly understood by them. Half knowledge is little better than none at all.

REMARK.—Conversation depends upon observation, memory, imagination, and discrimination.

4. Nothing would be lost, but considerable gained, if children had exercises in observation and conversation some weeks, or even months, before they begin to read.

5. All their lessons furnish material for conversation. No teacher need be at a loss for material, for he has the whole animal, vegetable, and mineral kingdoms from which to select. Judgment must, however, be exercised in the selection and in the order of presentation. The simplest and most interesting to children should come first, and then

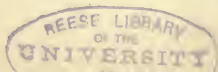
there should be a regular gradation in the order best adapted to the growth and development of their minds.

6. The most interesting objects for children are living things, animals. Those with which they seem best acquainted, and which they can the more readily examine and study, should come first. The cat, dog, cow, horse, pig, sheep; chickens, ducks, geese, turkeys, pigeons, parrots, canary-birds, are not only interesting but profitable objects for observation and study. Notwithstanding that we seem to be familiar with them, to our discredit it must be said, we know little about them. Children should be led to study them, and conversations about them will in the best way lead to this.

REMARK.—This kind of language work will require the teachers to familiarize themselves with the objects which the children are studying, and will thus at the same time enlarge their own sphere of knowledge.

7. Flies, bees, wasps, spiders, butterflies, bumblebees, all suitable kinds of insects, worms, birds, etc., found in the community, afford suitable material for observation, study, and conversation.

8. The vegetable kingdom should also be drawn upon. Roots, stems or stalks (or trunks), leaves, flowers, and seeds should be observed; their similarities and dissimilarities noticed, and their uses discussed. Plants about home, including grasses and trees, should first receive attention; and it cannot be too deeply impressed upon the minds of the teachers that the things themselves, and not pictures or descriptions of them, are to be the objects of observation and study. Plants may be kept in pots in the windows or other places of the school-room. Towards spring, twigs may be cut from trees, and, in jars partly filled with water, placed in the windows where they will be in the sun, and the opening of their buds observed from day to day. Seeds may be studied in the fall of the year. Towards



spring they may be planted in "shallow boxes, saucers, or any other convenient vessels filled with moist earth, sand, or saw-dust." They may also be planted on cotton and other suitable material floating upon water in wide-necked bottles, or they may be put into a sponge placed in a saucer of water. In this way their germination, the growth of the root, stem, etc., may readily daily be observed and noted.

9. The various kinds of minerals, including earths and stones, can also be made a profitable study, and therefore afford good material for conversation.

REMARK.—Jackman's *Nature Studies*, published by Henry Holt & Co., New York, is a book of valuable suggestions.

10. Conversations may be had about things in the school-room, on the school-grounds; appearance of the sky, kinds of clouds, direction of winds and their effect upon the weather. Things at home, such as may be found in the sitting-room, the parlor, on the table, in the kitchen, the laundry, the pantry, the cellar, the barn, on the farm, etc., may also be used.

11. The children should have opportunities to learn and to tell how the various kinds of business of a community are conducted. They should name the kinds of stores; tell what is sold in them; where and how the goods are procured; in what ways, where, and upon what terms they are bought; also upon what terms generally sold; and the necessity of such places of business. The post-office, express-office, bank, court-house, jail, railroad, railroad station, should be explained and discussed. Their advantages or necessities, as the case may be, should form part of the discussion. In the same way may the Sunday-school and the church be used and their influence upon the community be brought out. The various town and township officers, their necessity, duties, and how they are elected, installed, and paid, will at the same time be lessons in civil government.

REMARK.—What the children do not know the teachers must tell them.

12. Events of the day that are within the children's comprehension, and in which they can be interested, should be used for oral language work.

REMARK.—As will have been observed, the foregoing material for conversation is designed to make the children acquainted with their surroundings, and to make them observant, thoughtful, and intelligent.

II. WRITTEN LANGUAGE.

REMARK 1.—All the material suitable for oral language is equally so for written work ; and as soon as the pupils can use the pen or pencil they should be encouraged to talk with it, or, in other words, to write their thoughts, or at least some of them. Copying well written extracts and writing from memory, are good language exercises, occasionally to be used with beginners.

REMARK 2.—Besides the regular daily lessons in the various branches of study, which should all at the same time be regarded as lessons in language, the following exercises and suggestions, if intelligently used, will aid the teacher in training his pupils to the writing of good English.

1. Relate something of interest to them and let them reproduce it in their own words.

2. Read short, easy, but complete extracts, stories, or items of news, to them, and let them give the substance in their own words.

3. Let them say what they saw on their way to school, to the post-office, the store, the church, etc.

4. Let them write the history of an hour, a day, a week, a month, etc.

5. Let them write a narration of a walk, a ride, a real or an imaginary journey, a picnic, party, sociable, or anything that has occurred of which they have knowledge, or which they imagine as having occurred.

6. Let them describe a room, house, barn, stable, school-house ; the post-office, court-house, church ; a chair, table,

bench, door, window, knife, pair of scissors, thimble, looking-glass, clock, stove, wash-stand, comb, brush, sewing-machine, hat, cap, desk, tree, fence, gate, wagon, wheelbarrow, shovel, fork, hammer, broom, horse, cow, dog, cat, rat, mouse, pig, sheep, fly, spider, caterpillar, beetle, plant, flower; in short, anything that admits of description and that, in the judgment of the teacher, is not beyond their ability.

7. Let them compose (make) problems; also write out solutions of problems; sometimes all in words, without figures.

8. Let them name and describe articles of clothing and the materials of which they are made.

9. Let them write what they know of the school district, township, town, county, state, other states, countries, etc. These exercises, like the preceding, will serve the double purpose of knowledge and language.

10. Let them tell what they know of some of the most prominent or noted men of the country or world.

11. Let them state what they know of the history of the town, township, county, state, country, or anything else that has a history. The manufacture of an article from its beginning to its end or completion, may be written in the form of a history. The manufacture of a pin, needle, knife, pen, pencil, thimble, comb, cent, bank-note, hat, book, loaf of bread, pie, cake, window-pane, chair, table, sheet of paper, piece of cloth, etc., forms suitable subjects for this purpose.

12. Let them describe a picture, telling what they see, or imagine they see, in it, and arrange the description in proper order, so that any one on reading it or hearing it read could form a good mental picture of it.

13. Let them write letters to imaginary friends, telling them what they are doing, have done, or expect to do; or what they are learning about an object, a branch of study, a town, county, state, country, man, an event in history, etc.

14. Let them say as many things as they can about an object—a dog, cat, cow, horse, pig, sheep, table, chair, stove, door, window, clock, looking-glass, sofa, bureau, bench, knife, tree, apple, peach, pear, etc.; and lead them to see that some of the words are the same in all the lines they have written, and that all can be written in one line by using the repeated words but once.

REMARK.—These exercises will enable the teacher to introduce punctuation points and connecting words.

EXAMPLES OF 14:

- a. 1. The apple is large.
2. The apple is round.
3. The apple is red.
4. The apple is ripe.
5. The apple is soft.

Condensed.—The apple is large, round, red, ripe, *and* soft.

- b. 1. The apple is large.
2. The apple is round.
3. The apple is ripe.
4. The apple is not soft.

Condensed.—The apple is large, round, and ripe, *but* not soft.

- c. 1. The river is long.
2. The river is wide.
3. The river is not deep.

Condensed.—The river is long and wide, *though* (or *but*) not deep.

- d. 1. The needle is long.
2. The needle is thick.
3. The needle is not sharp.
4. The needle is not hard.

Condensed.—The needle is long and thick, *but neither* sharp *nor* hard.

15. Let the pupils say (write) the same thing of as many objects as they can, and then write all in one line, or sentence, with the repeated words used only once.

EXAMPLE OF 15:

1. The cat can run.
2. The dog can run.
3. The cow can run.
4. The horse can run.

Condensed.—The cat, the dog, the cow, and the horse can run; or the cat, dog, cow, and horse can run.

16. Incorrect expressions, such as are frequently heard in conversation, may be interspersed among the language exercises for correction—to teach the pupils the correct expression.

17. The following expressions are frequently heard, even from the lips of educated persons :

- a. Go and lay down. (Lay down what?)
- b. He laid on a sofa three weeks. (Laid what on a sofa?)
- c. I lay down every day an hour or two. (Lay down what?)
- d. He set on a log until noon. (Set what on a log?)
- e. He lays in bed too long. (Lays what in bed?)
- f. He comes here most every day. (Most day?)
- g. He thought it was us. (Us was?)
- h. It was me who said it. (Me was?)
- i. The river is raising. (Raising what?)
- j. He can do it as good as I can. (Do it good?)
- k. I know that it was them. (Them was?)
- l. If I was him I would not do it. (When? Him was?)
- m. It was her that done it. (Her was? Did she done it?)
- n. I wish that I was a musician. (When?)
- o. This is to be divided amongst you and I. (Amongst I?)
- p. A person must be stupid if they cannot do that. (A person they?)
- q. It could not have been her. (Her could not have been?)
- r. He told me that I can go. (How did he know?)
- s. Is that him? (Him is?)
- t. I expect you had a good time last night. (When do you expect it?)
- u. I wish that you had went earlier. (Why had went?)
- v. At what hotel are you stopping at? (Stopping what? At stopping at?)
- w. Who is your letter from? (From who?)
- x. Who are you writing to? (To who?)
- y. Each of the boys have their books. (Each their, and each have.)
- z. Leave me alone, I want to sleep. (Why alone you?)

18. The various kinds of sentences (according to their use) may now gradually be introduced. The statement should come first, next the question, then the command, and last the exclamation.

The statement is best taught in comparison with expressions that are incomplete, that do not assert anything. For

example : Some of the rivers of North America are long. This is a sentence, because it is a complete statement, a complete assertion ; but, *some of the rivers of North America*, is not a sentence ; it does not state or assert anything ; it is simply the beginning of a statement.

19. The following exercises will furnish a large amount of profitable language work :

a. 1. Write a request to your teacher to permit you to borrow a book from one of your classmates.

2. Ask him to permit you to go to the library to consult some books of reference ; state the information you desire, and in what books you expect to find it.

3. Ask permission to leave school after you have recited your last lesson, giving a full reason for your request.

4. Ask him to spend the evening at your house, say who else has been invited, that you expect a pleasant time, and that your parents desire his presence.

5. Tell him how you spend your evenings ; how much time you spend upon each lesson, the order in which you study your lessons, your methods of study, what recreations you have, and when you retire.

6. Ask him what the best way is to study each lesson ; whether it makes any difference in what order they are prepared, that is, which comes first, which next, and so on ; and say that you will thankfully receive any suggestions that he may make concerning your studies.

b. 1. Write to one of your acquaintances, asking what school he attends, whether public or private, what branches he studies, which he likes best, and how he likes the school.

2. Ask him what profession or trade he has in view, why he has selected it, which of the branches he is pursuing will be of most service to him in his life work, and why.

3. Tell him that you intend to spend next-Saturday afternoon at fishing, and that you would like to have him accompany you. Tell him what kind of lines and hooks he

will need, what kind of bait, where you intend to go, and where he shall meet you.

c. Ask one of your classmates or acquaintances to take a ride into the country with you ; name the objects of interest you expect to see ; speak of the enjoyment his company will afford you ; state that your father has given his consent for you to go, and that you will have a good horse and buggy.

d. 1. You are visiting one of your relatives in the country. Write a letter to your mother (if you are a girl) telling her of any incidents that came under your notice on your way, at the railroad station before starting, in the cars, or after you had left the cars. Tell her who met you at the station, and how you were received by the family—how glad they were to see you. Tell her, too, how you spend your time, whether you go out into the fields, and what you see when you do so.

2. Write to your sisters and tell them how you enjoy yourself ; tell them at what hours you rise and retire, how you spend your time before breakfast, what you do after breakfast, how the work of the family is done, how well it is systematized, what is assigned to each as a regular duty, and how well each performs his or her part.

3. Write to your father (if you are a boy) and tell him what you saw on the way to the station, how long you had to wait for the cars and why, whom you met at the station and what you learned from them, whom you saw on the cars, and anything of interest that may have occurred or come under your notice on the way.

4. Tell him the number of cattle the people you visit keep and the kinds, how many horses they have, the kind and number of vehicles, the farming implements and where they keep them, how they do their work, what crops they raise, the condition of the farm and buildings, how you are

enjoying yourself, and what new things you have seen and learned.

e. A salesman is wanted in a city clothing house. Write a letter to the proprietor asking for particulars concerning the position: 1. Whether any other duties would be required but that of being in the store during the day; 2. At what hour in the morning you would be required to begin work and at what hour in the evening close the store; 3. Whether any work would be required in the evening after closing; 4. What salary may be expected.

Write a reply to your letter as if it came from the proprietor, giving definite information on every point of inquiry, and if the reply is favorable—if the position suits you, write to the proprietor offering your services, stating your experience as a salesman and your ability to win customers and make sales. State, also, that, if acceptable, you are willing to be on trial a month or more. It would be of service to you, too, to state what educational advantages you have had, and that you can bring testimonials as to your moral character.

f. You have been invited to an evening party or entertainment at the house of a friend, but find it impossible to be present. Write a letter expressing your regret, and wishing your friend a pleasant time.

g. Write the autobiography of a pin, ball, shoe, nail, thimble; piece of paper, cloth, glass, bread, etc.

h. You have lost a valuable watch. Write a notice for the paper describing the watch, telling where you suppose you lost it, stating what reward you will give the finder on delivering it to you, and giving your place of business or residence—street and number.

i. You need an office-boy to run errands. Write an advertisement for the paper, stating definitely the kind of boy you want, what his duties will be, and compensation you will give.

j. You have received a valuable dog as a present. Write a letter to the person from whom you received it, thanking him for it, and telling him that you prize it highly, that you are fond of hunting, and this is the kind of dog you have for some time desired to get.

k. Write to a city publishing house for the prices of books which you need, and ask for the cheapest way of sending them—by mail or by express.

l. Mr. Hix wants to build. Write to him and tell him that you will take the contract at as reasonable a sum as any competent builder can do it, and that you will guarantee him a good job. Give references of persons for whom you have done similar work.

m. Write to a city dry-goods store for goods for a spring dress, stating the kind and quality you want, ask the prices, and say that if the reply you receive is satisfactory, you would like to know how to send the money.

Write a full, definite reply, and say that if you intrust your order to the "house" it shall receive prompt attention and be in all respects satisfactory.

Send the order, enclosing the money, and state how the goods shall be sent. After you have received the goods, acknowledge the receipt of it, and state how well you are pleased with your purchase, and that the house may expect further orders from you.

n. Samuel Poll owes you three hundred dollars, and you intend to purchase some property. Write him a letter stating that you contemplate making some investments, and that you should be glad if he would let you have the money by the first of next April.

Write his reply. He has been unfortunate during the year; the high waters destroyed his crops, carried away his fences, and caused an unexpected outlay of money. Besides, he has lost one of his best horses. He hopes, therefore, that you will give him more time.

Since he cannot pay you at the time you named, he should give you an interest-bearing note for the money. Write the note.

You need the money and can get it on the note by indorsing it. Indorse it in full.

o. Write a receipt for payment in full to date.

p. Write an order for six different kinds of goods. Receipt it.

q. Write a note of invitation ; also an acceptance of the invitation, stating the pleasure it will afford you to spend the afternoon and evening with the family and their friends.

Also write a note expressing your inability to accept the invitation, owing to sickness in your family, and express the desire that the family shall pay you a visit.

r. Write a due-bill, and state how the debt is to be discharged, whether with money or with goods. If with goods, name the kinds and quality.

s. Write a notice of meeting to be held, naming the place, the object, the day, and the hour.

t. Write an advertisement for the sale of household goods, stating the place, the day and hour, the articles to be offered for sale, and the conditions upon which they are to be sold.

For an additional number of good topics for Language and Composition, see *Our Language*, published by Leach, Shewell & Sanborn, 87 Franklin Street, Boston ; *Studies in English Composition*, published by Allyn & Bacon, Boston ; *Shaw's English Composition*, published by Henry Holt & Co., 29 West Twenty-third Street, New York.

20. The classes of words (parts of speech), of which the English language is composed, as has already been stated, should be taught in connection with reading and language exercises, and may be commenced in the Second Reader.

The simplest name should be applied to each class of words, a name, as far as possible, to which the children can

attach the proper meaning. The noun, for example, may be called the name-word ; the verb, the telling-word ; the adjective, the kind-word ; the preposition, the relation-word ; the adverb, the how, when, and where word (as the case may be) ; the conjunction, the connecting-word ; and so on.

21. The classification of sentences according to form into simple, complex, and compound, may gradually be introduced.

22. The various forms of phrases should be taught in connection with simple sentences, and those of clauses with complex. At the same time it may also be shown that phrases and clauses are substitutes for words, and enable us to give not only variety to our expressions, but frequently greater clearness.

23. A knowledge of the different forms of sentences, phrases, and clauses is best acquired by practice in writing them, substituting one form for another, and pointing out and discussing their various elements, relations, and uses.

REMARK.—As soon as the pupils can readily recognize the simpler grammatical distinctions, they may with some advantage begin to use a book on grammar for reference.

24. Substitute equivalent phrases for the underlined words in the following exercises :

- a. He passed my sister's house.
- b. She waited anxiously for the doctor.
- c. I could not hear him for the noise.
- d. The directors meet monthly.
- e. He occupies an influential position.
- f. They had erected a brazen image.
- g. Hence you will see the necessity of it.
- h. American ideas are liberal.
- i. The old oaken bucket had fallen to pieces
- j. The doctor is an intelligent man.

25. For the phrases in the following sentences substitute equivalent clauses :

- a.* The wool of the sheep affords us clothing.
- b.* None in the village suspected him of the deed.
- c.* With diligence he must succeed.
- d.* I will show you the place of my birth.
- e.* No one of my acquaintance was in the room.
- f.* I will tell you the reason of my acting so.
- g.* Do you know the age of the child?

26. Change the clauses in the following sentences to equivalent phrases :

- a.* I see no way in which I can improve it.
- b.* Any person who has good manners will be received.
- c.* His house stands where the battle was fought.
- d.* All stood with uncovered heads while he read the service.
- e.* When they heard the news they jumped for joy.

27. For the phrases in the following sentences substitute words of equivalent meaning :

- a.* He found it of benefit to use.
- b.* She calls upon them every day.
- c.* He selected it in place of his brother.
- d.* Your conduct was like that of a tyrant.
- e.* It would be of no use to try again.
- f.* They did it without the authority of law.

28. Make a simple sentence of each set of the following elements :

a. In America the railways are frequently single lines. The railways are formed to carry a limited commerce. Sidings are provided at convenient situations.

b. The river overflowed. The river was the Ohio. This was in January. It happened on the tenth of the month.

c. Wodin was the chief god of the Old English people. Wodin was by the Danes called Odin. He was the chief giver of valor. He was the chief giver of victory.

d. John Wycliffe did a great work. This great work was a translation of the Bible. The translation was made by himself. He was assisted by several friends and followers.

e. The English fearlessly boarded the ships. The ships were those of the enemy. They cut the rigging. They gained the victory. The victory was easily won.

f. The next morning the battle began in terrible earnest. The next morning was the 24th of June. The battle began at break of day.

g. Columbus returned to Spain in 1493. He had spent some

months in exploring the delightful regions. These regions had long been dreamed of by many. These regions were now first thrown open to European eyes. Columbus had been absent seven months and eleven days.

29. Combine each of the following sets of simple sentences into a complex sentence.

a. Tin is a metal. Ancient Britain was most famous for tin. The Phœnicians were first induced to visit Britain for tin.

b. He spoke to the king like a rough man. I think this myself. He was a rough angry man. He did nothing more.

c. The ingenuity of man has made a lever of the mind. This lever spares him an immensity of toil. This lever is applied to machinery.

d. The ships of Sesostris, the Egyptian conqueror, were formed of cedar. One of these ships was 280 cubits long. Ancient writers notice this. The gigantic statue of Diana in the temple of Ephesus was also formed of this timber.

e. Andrew Douglas was willing to share the danger and the honor. Andrew Douglas was master of the Phœnix. He had on board a great quantity of meal from Scotland.

f. Three or four bounds of the horse carried us out of reach of the enemy. The enemy quickly resumed his flight. The enemy had merely turned in desperate self-defence.

g. God in his good has covered the earth with herbs and trees. We inhabit the earth. These herbs and trees furnish us with food, clothing, and other articles. These articles contribute to our comfort and luxury.

h. At length the mystery of the ocean was revealed. The theory of the great navigator was triumphantly established. The theory had been the scoff of sages. He had secured to himself glory. This glory must be durable. The world itself is durable.

30. Change the following simple sentences to compound sentences :

a. The Rhone, flowing into the Lake of Geneva, emerged from it at the town of the same name.

b. These events, trifling doubtless in the estimation of the reader, were affecting to me in the highest degree.

c. Snatching the handkerchief, he quickly wrapped it around the wounded part.

d. The trees met overhead, forming an archway.

e. On further consideration I have decided to remain.

f. After a moment's reflection he proceeded to pass sentence.

g. The king, a man of rare vigor, would allow no foreign prince to encroach on his rights.

h. In forwarding your own interests, do not impede those of others.

i. The coral insect, barely possessing life, is hourly creating habitations for men.

31. Combine each set of the following simple sentences into a compound sentence :

a. They next erected a crucifix. They prostrated themselves before it. They returned thanks to God. God had conducted their voyage to such a happy issue.

b. He possessed quick perceptions. He observed accurately. He was able to place his hand on the right animals. He did so without hesitation.

c. The island at first seemed uninhabited. The natives gradually assembled in groups upon the shore. The natives gradually overcame their natural shyness. The natives received us very hospitably. They brought down for our use the various products of their island.

d. The struggle was now at an end. The inhabitants were terror-stricken. They burst through the lines. They fled in every direction.

e. They saw their leader fall. They thought him slain. They at once gave up the contest. This was in accordance with the practice of their ancestors.

f. Steam has increased indefinitely the mass of human comforts. Steam has increased indefinitely the mass of human enjoyments. Steam has rendered cheap the materials of wealth and prosperity. Steam has rendered accessible the materials of wealth and prosperity. It has done so all over the world.

g. The sun then broke out. The sun dispersed the vapor and the cold with his welcome beams. The traveller felt the general warmth. The sun shone brighter and brighter. The traveller sat down. The traveller was overpowered by the heat. The traveller cast his cloak upon the ground.

32. Change the following compound sentences to complex sentences:

a. You have asked me a question and I have answered it.

b. The statement is false and he knows it.

c. They did not know their lesson and so he kept them in.

d. Finish this and then I will let you go.

e. He was very ill, but still he tried to finish it.

f. A boy had seen it fall and had picked it up.

g. He tried several keys, but none of them would fit it.

For a greater variety of exercises in the various kinds of sentences, see First Steps and Second Steps in English

Composition, published by W. Stewart & Co., London, England ; Practical Exercises in Composition, and Exercises in English, by H. I. Strang, the former published by The Educational Publishing Co., Boston, the latter by D. C. Heath & Co., Boston. Kerl's Composition and Rhetoric is also a good book for sentence and composition work ; published by American Book Co., New York.

33. Exercises in the discrimination of words should form part of the sentence work.

Write sentences in which the following words shall be correctly used : Raise, rise ; sit, set ; bring, fetch ; fly, flee ; flow, flew, fled ; shut, close ; board, feed ; hung, hanged ; lay, lie ; leave, let ; lend, borrow ; lose, loose ; teach, learn ; wring, ring ; begin, commence ; forsake, desert ; load, burden ; empty, vacant ; sleigh, slay ; expect, suspect, suppose ; believe, calculate ; may, can ; fix, repair, mend ; think, guess ; enjoy, possess ; between, among ; invent, discover ; handful, handfuls ; preacher, minister, pastor, clergyman ; sow, sew ; luck, success ; stationary, stationery ; desert, dessert ; continual, continued ; composition, essay ; there, their ; hard, difficult ; balance, remainder ; advice, advise ; all, awl ; aloud, allowed ; altar, alter ; ant, aunt ; ascent, assent ; assistance, assistants ; bail, bale ; bait, bate ; bald, bawld ; bear, bare ; base, bass ; beat, beet ; blew, blue ; bawl, ball ; burrow, borough ; bough, bow ; cannon, canon ; capital, capitol ; ceiling, sealing ; cell, sell ; cent, sent, scent ; chord, cord ; site, cite, sight ; climb, clime ; council, counsel ; fare, fair ; gait, gate ; great, grate ; holy, wholly ; gage, gauge ; hail, hale ; pale, pail ; pane, pain ; plane, plain ; pray, prey ; rain, rein, reign ; sale, sail ; strait, straight ; vane, vain, vein ; pare, pear, pair ; canvass, canvas ; tacks, tax ; claws, clause ; nought, naught ; feet, feat, leave, lieve ; meet, mete, meat ; peel, peal ; pleas, please ; seed, cede ; seas, sees, seize ; heard, herd ; lesson, lessen ; miner, minor ; petition, partition ; of, off.

34. The properties that characterize well-written sentences, paragraphs, and essays, under the usual titles of purity, propriety, precision, and clearness, strength, unity, and harmony, should not, as is usual, be postponed until a late period of a pupil's school life, but should as early as is possible be introduced by directing his attention to meritorious as well as to faulty constructions, and demanding cor-

rections or improvements so far as he is capable of seeing their force and making them.

35. The only sure way of training pupils to the careful use of English is to begin as early as their years permit, and to demand that every exercise of theirs shall be as nearly perfect as they can make it ; and this course must be continued to the end of their school days.

36. Essays.—When pupils have acquired sufficient power of thought and skill in expression to write essays, subjects for the purpose may be assigned them. Care must, however, be taken that no abstract subjects, no subjects beyond their comprehension, nor anything in which they cannot be interested, be given.

37. After a general subject has been selected or assigned, it should be considered in all its bearings, and some special line of thought or view of it decided upon.

REMARK.—Writing upon general subjects cannot end in anything definite.

38. Having determined upon the line of discussion, the next thing is to keep it before the mind until it has been thoroughly thought over and everything found that has a direct bearing upon it. This done, a careful, logical outline of the main and sub-topics should be made, containing nothing not strictly in accord with the determined line of thought.

REMARK.—Finding the matter for the essay and making the outline constitute the most important and difficult part of the work.

39. Next comes the writing, the composition. A good plan to pursue is, to write a little essay, as it were, upon each main topic, combine them into one, look it over to correct errors, then lay it away for a few days or a week before re-examination and rewriting. An essay should be several times carefully examined and rewritten before it is handed to the teacher for inspection and suggestions.

40. Unless intended for a public audience, no essay handed to the teacher should be corrected by him ; only the place where an error exists should be indicated by some general mark or sign, and the pupil, at least at first, left to discover the fault himself. It is only by practice in discovering that discovering is learned.

41. An essay should be handed two, three, or more times to the teacher for inspection and criticism, and as many times rewritten. In short, it should be criticised and rewritten until it is free from errors.

In few things, if in any, do we find more failures in teaching than in composition, and the *best* book on essay writing is a *competent* teacher.

The following books on composition, in addition to those already named, may be used with advantage by teachers : Longmans' School Composition, published by Longmans, Green & Co., New York ; The Foundations of Rhetoric, published by Harper & Brothers, New York ; Composition and Practical English, by William Williams, published by D. C. Heath & Co., Boston. For lower grades of schools, the following will prove serviceable : Stories for Composition, published by Educational Publishing Co., Boston ; How to Write a Composition, published by Dick & Fitzgerald, New York ; Primary Reproduction Stories, and Hall's Composition Outlines, published by A. Flanagan & Co., Chicago ; The Writing of Compositions, published by E. L. Kellogg & Co., New York ; and W. B. Powell's whole series of language books, published by E. H. Butler & Co., Philadelphia.

Nearly all the foregoing books can be had of E. L. Kellogg & Co., 61 East Ninth Street, New York.

SUGGESTIONS FOR TEACHING NUMBERS.

TEACH the concept (idea) concretely, with pebbles, beans, grains of corn, shoe-pegs (colored or plain), spools, squares, cubes, balls (spheres), cylinders, triangles; in short, with any suitable objects that may be had. A variety of objects should be kept on hand for this purpose.

Figures should not be introduced until the children can work well with objects, pictures, etc., and until they will not confound figures with numbers.

Color and form should be taught in connection with numbers.

LESSON ON TWO.

a. What the Pupils must Discover.

- | | | |
|-------------------|-------------------------|--------------------------|
| 1. $1 + 1 =$ | 5. $2 - 2 =$ | 9. $\frac{2}{2}$ of 1 = |
| 2. $1 \times 2 =$ | 6. $2 \div 1 =$ | 10. $\frac{1}{2}$ of 2 = |
| 3. $2 \times 1 =$ | 7. $2 \div 2 =$ | 11. $\frac{2}{2}$ of 2 = |
| 4. $2 - 1 =$ | 8. $\frac{1}{2}$ of 1 = | 12. $\frac{2}{2} =$ |

b. For seat work the following notation may be used with children that are not far enough advanced to write words or to use figures :

- | | | |
|-----------------------|------------------------------------|---|
| 1. $1 + 1 = 11$ | 5. $11 - 11 = 0$ | 9. $\frac{2}{2} (1) = \frac{2}{2} = 1$ |
| 2. $1 \times 11 = 11$ | 6. $11 \div 1 = 11$ | 10. $\frac{1}{2} (11) = \frac{1}{2} + \frac{1}{2} = 1$ |
| 3. $11 \times 1 = 11$ | 7. $11 \div 11 = 1$ | 11. $\frac{2}{2} (11) = \frac{2}{2} + \frac{2}{2} = 11$ |
| 4. $11 - 1 = 1$ | 8. $\frac{1}{2} (1) = \frac{1}{2}$ | 12. $\frac{2}{2} = 1$ |

NOTE.—1. The teacher should substitute some form of the concrete for the fractions in the seat-work, until the children can use figures. A short vertical line divided into two equal

parts, with a nought covering the upper part, might represent $\frac{1}{2}$; both parts uncovered, $\frac{2}{2}$; the line divided into three equal parts, with the upper part covered, $\frac{2}{3}$; with the two upper parts covered, $\frac{1}{3}$, etc.

2. Horizontal lines, squares, triangles, circles, and pictures of suitable objects may be used by the children in performing the fractional work; the whole object or picture representing or being the unit, and the parts the fractions.

3. In solving problems in which a fractional part is required of a number consisting of several ones, or units, the children should first be taught to find the sum of the fractional parts of the separate units; later, after they can readily do this, they should be taught the usual way of obtaining the result. Problem 10 of the foregoing seat-work may be solved with two horizontal lines, one of the halves of each being covered with a nought or some other device to indicate that it is not to be counted. It may also be solved with squares, as follows:

$$\begin{array}{|c|} \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} - \begin{array}{|c|} \hline \\ \hline \end{array}.$$

c. Suggestive Questions and Problems.

1. Give me a cube. Give me another. How many have I now? (One and one are called two; or, simply, are two.)

2. Show me two fingers, two hands, two boys, a two of red cubes, a two of pencils, a two of anything else.

REMARK.—A two of anything means two things taken together and considered as a unit.

3. One bird and one bird are how many birds?

4. Show me a two. Show me a one. How many ones are in a two?

5. If two birds are on a tree and one of them flies away, how many remain?

6. What number is one more than one?

7. What number is one less than two?

REMARK.—Every operation should be proved or illustrated with objects or drawings of them.

8. How many twos are in two ones? How many ones does it take to make a two?

9. From a two take away two ones, and what have you left? Prove it with yellow cubes.

10. From two ones take away a two, and what have you left? Prove it with blue cylinders.

d. Suggestive Dialogue to Teach the Half.

Teacher. If you wanted to give me half of an apple, how would you cut (or divide) the apple?

Pupil. I would cut it through the middle.

T. Which one of us would get the larger piece?

P. Neither; one piece would be as large as the other.

T. Make a picture-apple upon the blackboard, and draw a line through it where you would cut it.

T. Here are two pieces of cardboard; which of them is the longer?

P. One is as long as the other.

T. How much longer are the two pieces together than this piece?

P. They are just as long.

T. If I should cut the long piece across the middle, which of the two parts would be the longer?

P. Neither piece would be longer than the other.

T. What part of the whole piece would each of the parts be?

P. One half.

T. What did I do?

P. You cut the long piece through the middle.

T. How can you tell that I cut it through the middle?

P. By trying whether one piece is as long as the other.

T. You may try it.

P. One piece is just as long as the other.

T. What part of the whole piece do I hold in my hand?

P. One half.

T. One half of what?

P. Of the whole piece.

T. Lay one piece at the end of the other, and see what the two halves make.

P. They make the whole piece.

T. Since these two pieces make the whole piece, one of them is what part of two?

P. One half.

T. How do you know that each piece is one half of the two pieces?

P. Because it is one half of the whole piece, and the whole is made of the two pieces.

T. Which of these two yellow cubes is the larger?

P. One is as large as the other.

T. How do you know?

P. I have tried them.

T. Now since one of the two pieces of cardboard is one half of the two pieces, what part do you think one cube is of the two cubes?

P. One half.

T. One apple is what part of two apples?

P. If they are of the same size, it is one half of them.

T. How many halves are in the whole of anything?

P. Two.

T. How can I get one half of anything—of an orange, for example?

P. By cutting it through the middle.

T. Why cut it through the middle?

P. To make one piece as large as the other.

T. Why must one piece be as large as the other?

P. If they were not of the same size they would not be halves.

T. How can I get one half of two candies?

P. By breaking each one into two equal pieces, and taking a piece of each of them.

T. Could you get a half of the two in any other way?

P. If one of the candies is as large as the other, one of them would be a half of the two.

T. Which one of them?

P. Either one.

T. How can you tell?

P. Because one is as large as the other.

T. Are all halves of the same size?

P. Yes, they are.

T. You may draw two picture-apples upon the black-board, making one larger than the other.

T. Draw a line through the middle of each, and then tell me whether the halves of the small one are as large as those of the large one.

P. No; they are not.

T. Do you still think that all halves are of the same size?

P. No; I do not.

T. What halves, then, are of the same size?

P. Those of the same thing, or of things of the same size.

T. In how many ways could you cut this card (parallelogram, 1 in. by 2 in.) into halves?

T. I will give you a card and you may draw a pencil-mark across where you would cut it, and then tell me in how many ways you could cut it?

P. I could cut it in two ways, lengthwise and crosswise.

T. I will give you another card, and you may cut one of them lengthwise and the other crosswise, and then tell me whether all the halves are of the same size?

P. No, they are not; the pieces of the card cut lengthwise are longer than those of the one cut crosswise.

T. Do you notice any other difference?

P. Yes, the short pieces are broader than the others.

T. Find out how much broader they are?

P. They are twice as broad.

T. Now cut one of the short pieces into two equal pieces and see whether the two parts laid together make a piece as long as one of the long pieces.

P. Yes, they do.

T. What can you again say of the halves of anything?

P. That one is as large as the other.

T. In how many ways did you cut your card into halves?

P. In two ways.

T. What are they?

P. Lengthwise and crosswise.

T. Can you cut them in any other way so that the two parts will be of the same size?

P. I think I can, but I am not sure of it.

T. You may try it.

P. Yes, I can cut them in another way—I can cut them from one corner across the middle to the opposite corner.

T. How do you know that the pieces are of the same size?

P. I laid one upon the other and it covered it exactly.

T. Show me half of your cubes.

T. Of how many do you show me a half?

P. Of two.

T. How many halves does it take to make the whole of anything?

P. Two.

T. Two what?

P. Two halves.

T. To make what?

P. To make the whole of anything.

e. Suggestive Dialogue to Teach the Applications in Two.

T. What do I hold in my hand?

P. A two-cent piece.

T. How many cents would you give me for it?

P. Two.

T. How many cent-candies could you get for one cent?

P. One.

T. Illustrate (show) it with pictures upon the black-board; also with toy-money and crayons.

T. How many cent-apples could you get for two cents?

P. Two.

T. Prove (show, illustrate) it with picture-apples and picture-cents.

T. If I should send you to the store to buy two slate-pencils that cost a cent each, how much money would you pay for them?

P. Two cents.

T. Your sister sends you to the store with a two-cent piece to buy an orange that costs a cent; how much money will you bring back?

P. One cent.

T. If you should go to the post-office with two cents to buy cent-stamps, how many would you get?

P. Two.

T. Henry has two rabbits, and this is one more than Sarah has; How many has Sarah?

P. One.

T. John has half as many roses as his sister; if he has one, how many has she?

P. Two.

T. I know a number whose double is two, what is it?

P. One.

T. What number is that whose half is one?

P. Two.

LESSON ON THREE.

a. What the Pupils must Discover.

1. $2 + 1 =$

2. $1 + 2 =$

3. $1 \times 3 =$

4. $3 \times 1 =$

5. $3 - 1 =$

6. $3 - 2 =$

7. $3 - 3 =$

8. $3 \div 3 =$

9. $3 \div 1 =$

10. $3 \div 2 =$

11. $\frac{1}{3}$ of 1 =

12. $\frac{2}{3}$ of 1 =

13. $\frac{3}{3}$ of 1 =

14. $\frac{1}{3}$ of 2 =

15. $\frac{2}{3}$ of 2 =

16. $\frac{3}{3}$ of 2 =

17. $\frac{1}{2}$ of 3 =

18. $\frac{2}{2}$ of 3 =

19. $\frac{1}{3}$ of 3 =

20. $\frac{2}{3}$ of 3 =

21. $\frac{3}{3}$ of 3 =

REMARK.—1. In $3 \div 2$, the question is, how many twos are in three; and the answer is, one two and one one, and may be written thus, 1 (1), the parenthetic part being the remainder.

2. The yard with its parts in feet should be taught in connection with the foregoing.

b. For seat-work the following will serve as example :

- | | | |
|---|--|-----------------------|
| 1. $II + I = III$ | 4. $III \times I = III$ | 7. $III - III = 0$ |
| 2. $I + II = III$ | 5. $III - I = II$ | 8. $III + III = I$ |
| 3. $I \times III = III$ | 6. $III - II = I$ | 9. $III \div I = III$ |
| 10. $III \div II = I(I)$ | 16. $\frac{3}{3} (II) = \frac{3}{3} + \frac{3}{3} = II$ | |
| 11. $\frac{1}{3} (I) = \frac{1}{3}$ | 17. $\frac{1}{2} (III) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = I\frac{1}{2}$ | |
| 12. $\frac{2}{3} (I) = \frac{2}{3}$ | 18. $\frac{2}{3} (III) = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = III$ | |
| 13. $\frac{3}{3} (I) = \frac{3}{3} = I$ | 19. $\frac{1}{3} (III) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = I$ | |
| 14. $\frac{1}{3} (II) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ | 20. $\frac{2}{3} (III) = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = III$ | |
| 15. $\frac{3}{3} (II) = \frac{3}{3} + \frac{3}{3} = I\frac{1}{3}$ | 21. $\frac{3}{3} (III) = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = III$ | |

NOTE.—The method of solution is indicated in all the exercises that follow the tenth.

REMARK.—1. As before remarked, instead of the foregoing notation, horizontal lines, squares, triangles, circles, and pictures may be used for performing all the fractional work. For examples, the 20th of the foregoing may be solved thus, with squares :

$$\square\square + \square\square + \square\square = \square\square = \square\square \quad \square \quad \square$$

REMARK.—2. The pupils should be required to make stories of their exercises. Of No. 1 of the foregoing the following may be made : I had two cents and my sister gave me another; then I had three. Or, Sarah had two roses and her mother gave her another; how many had she then?

c. Suggestive Questions and Problems.

1. Give me two red spheres (balls). Give me another. How many have you given me altogether? (Two and one are three).

2. Show me three fingers, three cylinders, three girls, three boys.

3. What number is one more than two?

4. Three is one more than what number?

5. One is two less than what number ?
6. What number is less than three ?
7. What number is two more than one ?
8. To what number must I put (add) one to make three ?
9. One and one and one are how many ones ?
10. Show me a two, also two ones ; which is the larger ?
11. How many ones are in a two ? In a three ?
12. How many twos are in a three, or in three ones ?
13. Make three in all the ways you can.
14. Make three picture-boys on your tablet, three picture-girls, three picture-horses, three picture-wagons, three picture-cats.
15. Under each of three trees John found an apple; how many apples did he find ?
16. Three mice were in a box and Charles killed all but two ; how many did he kill ?
17. Henry had three roses and gave all but one to Alice; how many did Alice receive ?
18. Frank has three horses and one saddle, how many more horses has he than saddles ?
19. Ella had three playmates and gave to each a pear ; how many pears did she give away ?
20. Jacob caught three rabbits in three traps ; how many did he catch in each ?
21. If you should lay three grains of corn upon the floor and a mouse should come and carry one of them away at a time, how many trips would it have to make to carry all of them away ?
22. Place three grains of corn upon the floor ; now, if a mouse could carry away only one grain at a time, how many mice would be required to carry all of them away at once ? Make picture-mice and prove it.
23. Place three buttons upon the table ; take them away two at a time ; how many times did you take two away ?

24. How many twos in three? How many ones? How many threes?

25. Make two in all the ways you can.

26. Make three of ones; how many does it take?

27. How many ones in two? In three? How many twos in two?

28. How many ones in one half of two? In three halves of two?

29. How many twos in two ones? In three ones?

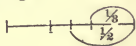
30. Take three buttons and lay each one by itself; how many separate buttons have you laid? Count them. Each one of the three is what part of all of them? (It is one of the three equal parts, or one third.)

31. Show me one third of three yellow squares; two thirds of three buttons; three thirds of three picture-apples.

32. How can you get one third of anything? Can you get one third of one? How? Do it.

33. How would you get one half of anything?

34. Which is the larger (or greater), one half or one third? Prove it.



35. How many halves in one and one half?

36. If I cut an apple into two equal pieces, what is one of the pieces called? What is each of the pieces called?

37. Show me how you would get two thirds of anything. Two is what part of three? One is what part of two?

38. This stick (3 inches) is how many times as long as that (1 inch)? How can you find out? Do it.

39. This stick (1 inch) is what part of that (3 inches). How can you tell?

40. This stick (1 inch) is what part of that (2 inches.)?

d. Suggestive Practical Business Problems.

We will now play store. John may be the merchant and the others his customers or buyers.

1. Sarah may buy two buttons, at a cent apiece, and give him a three-cent piece for them. How much change will he give her?

REMARK.—The more these transactions are made like real business the more interest the children will take in them.

2. Alice may buy a one-cent candy and a two-cent candy. How many cents must she give for them?

3. Henry buys three pencils at a cent each; how much money will pay for them?

4. Fannie wants three cents' worth of eggs, and the eggs cost a cent each; how many will she get? How many would she get for two cents? Illustrate.

5. How many two-cent rings can I get for three cents?

6. How many nuts at a half-cent each could you get for one cent? How many for a cent and a half?

7. Fill this pint measure with sand and pour it into that quart measure. Did it fill it? Fill it again and pour it in. Is it now full? What did I call this measure? What that? How many times this did it take to fill that? How many pints in a quart? In a half-quart?

8. One pint is what part of a quart? Two pints are what part of a quart? We will call this water milk, and Annie may sell it at a cent a pint.

9. Alice wants a quart and gives Annie a three-cent piece; how much change will she receive?

10. Sarah wants two pints; how much will they cost?

11. Jacob wants a half-quart and has a two-cent piece with which to pay for it; give him the change.

12. John may take this foot-measure and see how many such could be made of that yard-measure.

13. Henry, you may tell us how many foot-measures it takes to make a yard-measure, or a yard. Prove it.

14. How many feet make a yard, or are in a yard? How many in one third of a yard? In two thirds? In one half? Sarah may now sell tape to the other members of the class.

REMARKS.—Let the children make tape of newspapers, by cutting them into narrow strips and sewing the ends together.

15. Alice buys a yard that costs two cents and a yard that costs one cent ; how much will it all cost ?

16. Anna wants a third of a yard of that which costs three cents a yard ; how many cents must she give for it ? Prove it ?

17. Fannie wants a cent's worth of that which costs three cents a yard ? How much will she get ?

18. Peter wants a yard and a half of that which sells at two cents a yard ; how much money will pay for it ?

19. Ella had three yards and sold one third of them ; how many had she left ? If it sold at three cents a yard, what did she get for what she sold ?

20. When a quart of milk costs two cents, what does a pint cost ?

21. How many pints in a quart ? How many quarts in three pints ?

22. How many feet in a yard ? In a half-yard ? In a third of a yard ?

23. One foot is what part of a yard ?

24. If a yard of tape costs three cents, what does one foot of it cost ? What two thirds of a yard ?

LESSON ON FOUR.

a. What the Pupils should Discover.

1. $3 + 1 =$	9. $4 - 4 =$	17. $\frac{3}{4}$ of $4 =$	25. 1 qt. =
2. $1 + 3 =$	10. $4 \div 2 =$	18. $\frac{1}{4}$ of $1 =$	26. $\frac{1}{2}$ gal. =
3. $2 + 2 =$	11. $4 \div 1 =$	19. $\frac{2}{4}$ of $1 =$	27. 4 gills =
4. $2 \times 2 =$	12. $4 \div 3 =$	20. $\frac{3}{4}$ of $1 =$	28. $\frac{1}{2}$ pt. =
5. $1 \times 4 =$	13. $4 \div 4 =$	21. $\frac{1}{4}$ of $1 =$	29. 4 pk. =
6. $4 - 1 =$	14. $\frac{1}{2}$ of $4 =$	22. $\frac{1}{2}$ of $\frac{1}{2} =$	30. $\frac{1}{2}$ bu. =
7. $4 - 2 =$	15. $\frac{1}{3}$ of $4 =$	23. 4 qts. =	31. 1 pk. =
8. $4 - 3 =$	16. $\frac{2}{3}$ of $4 =$	24. 2 qts. =	32. 4 weeks =

b. The following will serve as examples of what may be given as seat-work :

- | | | |
|----------------------|----------------------------|---------------------------------------|
| 1. III + I = | 10. IIII + II = | 18. $\frac{1}{4}$ (I) = |
| 2. I + III = | 11. IIII + I = | 19. $\frac{2}{4}$ (I) = |
| 3. II + II = | 12. IIII + III = | 20. $\frac{3}{4}$ (I) = |
| 4. II $\times$ II = | 13. IIII + IIII = | 21. $\frac{4}{4}$ (I) = |
| 5. I $\times$ IIII = | 14. $\frac{1}{2}$ (IIII) = | 22. $\frac{1}{2}$ (IIII) = |
| 6. IIII - I = | 15. $\frac{1}{3}$ (IIII) = | 23. $\frac{2}{4}$ (IIII) = |
| 7. IIII - II = | 16. $\frac{2}{3}$ (IIII) = | 24. $\frac{3}{4}$ (IIII) = |
| 8. IIII - III = | 17. $\frac{3}{4}$ (IIII) = | 25. $\frac{1}{2}$ ($\frac{1}{2}$) = |
| 9. IIII - IIII = | | |

NOTE.—In No. 25, the question is, What part of one is one half of one half of one? and by dividing each of the two halves of

a given line or square into two equal parts, thus, 

or , the answer is found to be one fourth.

REMARK.—The pupils should also be required to write the foregoing exercises in words, either in the question or in the statement form or in both. This work will give them practice in penmanship, spelling, and language, in addition to that of numbers. Nos. 1, 10, 14, 15, and 24, for example, may be written as follows: (1.) Three and one are how many? or three and one are four. (10.) How many twos are in four? (14.) One half of four ones (or of four) is two ones (or two). (15.) What is one third of four ones? (24.) What are three fourths of four ones, or of four (ones being understood)?

c. Suggestive Questions and Problems.

1. Take a one and a two. How many ones have you taken? How many threes?

2. Take a three and a one. How many ones did you take? (Three and one are four.)

3. Give me a two. Give me another. How many ones did you give me? How many ones are in four? How many twos? How many threes? How many fours?

4. Show me one four, one three, one two.

5. How many twos in three?

6. One half is what part of one and one-half? Of two ones?

7. How many ones in three halves? In four halves?

8. How many feet in a yard? In a half-yard? In two thirds of a yard?

9. How many pints in a quart? In a half-quart?

10. How many quarts will fill this gallon measure? How can you find out? Do it. We will call this water milk, and sell it at four cents a gallon.

11. Sarah wants a quart and gives a two-cent piece to pay for it? how much change shall she receive?

12. Alice wants a half-gallon; how many cents will pay for it?

13. Jacob buys two pints; what will they cost?

14. Anna wants two half-gallons; how much money will she need to pay for them? How many two-cent pieces will pay for them?

15. Here is a measure we have not yet used; it is called a peck, and is used for measuring apples, peaches, nuts, corn, wheat, etc., things not liquid and not sold by the pint or quart. Henry may fill it with saw-dust and empty it into that half-bushel. Does it fill it? Peter may fill it and also empty it into the half-bushel. Is the half-bushel now full? How many pecks make (fill) a half-bushel? How many a bushel (whole bushel)?

We will now sell peaches, at a dollar a bushel. You may get the money (toy-money). John may sell them. He has no peaches, but he may make believe (pretend) that he is selling and you may make believe that you are buying.

16. Jacob wants two pecks. He may show us the money he will give for them. How many quarter-dollars has he? Has he enough money? Prove it. How many half-dollars would pay for them? Prove it.

17. Peter sells pears, at two dollars a bushel, and John

buys a peck. He may get the money to pay for them. Did he get enough?

18. Jacob wants a bushel and a peck of pears. He may get the money to pay for them. Has he enough money? How can you tell?

19. Three quarters of a bushel make how many half-bushels? How many pecks?

20. How many half-dollars will pay for three quarters of a bushel of pears? What part of a dollar would pay for them?

21. Alice wants a bushel and a half; how much money will pay for them? Prove that your answer is right.

22. Sarah sells Henry two and a half yards of cloth, at one dollar a yard; how many dollars will pay for them? How many dollars will three half-yards cost? how many three quarters of a yard?

23. What number diminished by two leaves nothing?

24. From what number can I take one-half and have one and one fourth left?

25. From what number can I take two and one half and have one and one half left?

26. What number added to one makes four?

27. What number added to one and one half makes three?

28. I think of a number whose half is two; what is it?

29. What number taken four times makes four?

30. I think of a number whose fourth part is one; what is it?

31. What two numbers make four? What three numbers?

32. What number taken three times, and one added, makes four?

33. What number doubled makes four? What three?

34. What number doubled, and one added, makes three?

35. Make four in all the ways you can, mentally.

36. Make two in all the ways you can ; also three.
 37. What is one half of one ? of three ?
 38. What is one third of one ? of two ? of four ?
 39. What are two thirds of one ? of two ? of four ?
 of three ?
 40. What are two fourths of one ? Three fourths of
 one ? Three fourths of two ? of three ? of four ?

REMARK.—Whenever it is possible stories should be made of abstract problems. Stories give reality and interest to the work.

LESSON ON FIVE.

- | | | |
|-----------------------|--------------------------|--------------------------|
| a. 1. $4 + 1 =$ | 15. $5 \div 2 =$ | 29. $\frac{4}{5}$ of 2 = |
| 2. $1 + 4 =$ | 16. $5 \div 3 =$ | 30. $\frac{1}{5}$ of 3 = |
| 3. $3 + 2 =$ | 17. $5 \div 4 =$ | 31. $\frac{2}{5}$ of 3 = |
| 4. $2 + 3 =$ | 18. $5 \div 5 =$ | 32. $\frac{4}{5}$ of 3 = |
| 5. $2 + 2 + 1 =$ | 19. $\frac{1}{2}$ of 5 = | 33. $\frac{1}{5}$ of 4 = |
| 6. $2 \times 2 + 1 =$ | 20. $\frac{1}{3}$ of 5 = | 34. $\frac{2}{5}$ of 4 = |
| 7. $1 \times 5 =$ | 21. $\frac{2}{3}$ of 5 = | 35. $\frac{3}{5}$ of 4 = |
| 8. $5 \times 1 =$ | 22. $\frac{1}{4}$ of 5 = | 36. $\frac{4}{5}$ of 4 = |
| 9. $5 - 1 =$ | 23. $\frac{3}{4}$ of 5 = | 37. $\frac{1}{5}$ of 5 = |
| 10. $5 - 2 =$ | 24. $\frac{1}{5}$ of 1 = | 38. $\frac{2}{5}$ of 5 = |
| 11. $5 - 3 =$ | 25. $\frac{3}{5}$ of 1 = | 39. $\frac{3}{5}$ of 5 = |
| 12. $5 - 4 =$ | 26. $\frac{5}{5}$ of 1 = | 40. $\frac{4}{5}$ of 5 = |
| 13. $5 - 5 =$ | 27. $\frac{1}{5}$ of 2 = | 41. $\frac{5}{5}$ of 5 = |
| 14. $5 \div 1 =$ | 28. $\frac{3}{5}$ of 2 = | |

b. Examples of Seat-work.

REMARK.—For the seat-work of *five* and the following numbers, until the children can use figures, the notation of the numbers preceding five, or the following, may be used.

- | | |
|-----------------------------|--------------------------|
| 1. four + one = | 12. five - four = |
| 2. one + four = | 13. five - five = |
| 3. three + two = | 14. five + one = |
| 4. two + three = | 15. five + two = |
| 5. two + two + one = | 16. five + three = |
| 6. two $\times$ two + one = | 17. five + four = |
| 7. one + two $\times$ two = | 18. five + five = |
| 8. one $\times$ five = | 19. one half of five = |
| 9. five - one = | 20. one third of five = |
| 10. five - two = | 21. two thirds of five = |
| 11. five - three = | 22. one fourth of five = |

- | | |
|-----------------------------|----------------------------|
| 23. three fourths of five = | 33. one fifth of four = |
| 24. one fifth of one = | 34. two fifths of four = |
| 25. three fifths of one = | 35. three fifths of four = |
| 26. five fifths of one = | 36. four fifths of four = |
| 27. one fifth of two = | 37. one fifth of five = |
| 28. three fifths of two = | 38. two fifths of five = |
| 29. four fifths of two = | 39. three fifths of five = |
| 30. one fifth of three = | 40. four fifths of five = |
| 31. two fifths of three = | 41. five fifths of five = |
| 32. four fifths of three = | |

c. Suggestive Questions and Problems.

1. Place four blocks upon the table. Place another on. How many are on now? (You have placed five on.) How many did you place on first? How many afterwards? How many altogether? Then four and one are how many?

2. Can you make five of ones? Try it. How many does it take?

3. Can you make five of twos? How many does it take? Then two twos and one are how many? How many ones?

4. Can you make five of threes? How many does it take? One three and one two are how many? One three and two ones are how many?

5. Can you make five of fours? How many does it take?

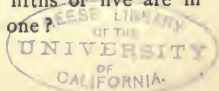
6. Make five in all the ways you can (in your mind) without objects, and in every case tell the result.

7. How many threes are in four? How many twos? How many ones?

8. What is one half of four? of two? of one? One third of three? of four? of one?

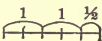
9. What is one fourth of four? of three? of two? of one? One third of two?

10. Divide five pebbles into five equal parts. How many have you in each? One is what part of five? How many such parts are in five? How many fifths of five are in five? How many fifths of one are in one?

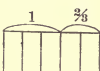
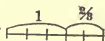


11. How many times can you find two fifths of one in one?

REMARK.—Problems of this kind should at this stage of the pupil's progress be solved with diagrams. Solution of prob-

lem 11:  , or  The answer is $2\frac{1}{5}$.

12. How often can you find three fifths of one in one?

Solution:  , or  = $1\frac{2}{5}$.

13. How often can you find four fifths of one in one? Prove it.

14. How often can you find two thirds of one in one?

Prove it. Solution:  , or  = $1\frac{1}{2}$.

15. What is one half of five? One third of five? One fourth of five? One fifth of five? Solution of first:

$$\overbrace{\frac{1}{2} + \frac{1}{2}}^1 + \overbrace{\frac{1}{2} + \frac{1}{2}}^1 + \frac{1}{2} = 2\frac{1}{2}.$$

16. What are two thirds of five? Three fourths of five? Two fifths of five? Solution of first:

$$\overbrace{\frac{2}{3} + \frac{2}{3}}^1 + \overbrace{\frac{2}{3} + \frac{2}{3}}^1 + \overbrace{\frac{1}{3} + \frac{1}{3}}^{1\frac{1}{3}} = 3\frac{1}{3}.$$

17. I know a number to which if I add three it will make five; what is it?

18. I know a number that contains two twos and one one; what is it?

19. I think of a number to which if I add four it will make five; what is it?

20. I know a number whose half is two; what is it?

21. I know a number whose half added to it makes three; what is it?

22. From what number can I take its third and have two left?

23. If to a certain number I add its two thirds, the sum will be five; what is the number?

24. What number added to one and one half makes four?

25. I think of a number that is three less than five; what is it?

26. What number is two less than three?

27. If from a certain number I take its fourth, three will be left; what is it?

28. What number doubled makes five?

29. Three times a number and twice the number make five; what is the number?

30. What number doubled and its half added makes five?

31. What number increased by its fourth equals five?

32. What number lacks two of being five?

33. What two unlike numbers make five? What unlike numbers make four? What like numbers?

34. I buy apples at two cents each and they cost me four cents; how many do I buy?

35. I bought candies at one cent each, and out of three cents received one cent change; how many did I buy?

36. I paid four cents for two pencils; how much did they cost me apiece?

37. I bought two-cent oranges, and out of five cents received one cent change; how many did I buy?

38. Sarah had five two-cent pieces and gave one to each of three boys; how many had she left? How many cents could she get for them?

39. A bird laid five eggs and hatched them all but four; how many did it hatch?

40. Two boys and three girls went coasting; the number of boys is what part of that of the girls? By what part of

their number should that of the boys have been increased to have made it equal to that of the girls?

41. Jane has four ducks and three pigeons; how many times as many ducks has she as pigeons? The number of pigeons is what part of that of the ducks?

42. Henry's book cost him three cents, and his pencil one third as much; how much did both cost? The difference in cost of the two is what part of the cost of the book? The cost of the pencil is what part of the difference?

43. One is what part of two? of three? of five?

44. Two is what part of three? of four? of five?

45. Three is how many times two? Three is what part of three? of four? of five?

46. Four little birds were in a nest and one half of them flew away; how many remained?

47. Three boys are playing ball; what part of their number must be added to them to make it five?

48. How many two-cent postage-stamps can you get for five cents?

49. How many yards of tape, at three cents a yard, can you get for five cents? What part of a yard for two cents?

50. If your mother should send you to the store with five cents, to buy thimbles that cost four cents each, how many could you get? How much change would you get?

51. With five cents buy all the two-cent lemons you can? How many can you get? Prove it.

52. A bushel of apples costs three quarters of a dollar; how many bushels can you get for two dollars? for three dollars? For two and one half dollars?

53. A bushel of pears costs two dollars; how many bushels could you get for five dollars? for two and a half dollars? What part of a bushel could you get for one and a half dollars? for three fourths of a dollar? for a half dollar? How many pecks could you get for one and one fourth dollars?

54. A peck of plums costs a half dollar ; how many half bushels could you get for a dollar and a half. How many bushels for three dollars ?

55. When a half bushel of quinces costs one and one half dollars, how much does a peck cost ? how much a bushel ? What part of a bushel could you get for a half dollar ? for two dollars ?

56. When a gallon of honey costs two dollars, what part of a gallon can be had for a half dollar ? for a dollar ? How many quarts for a dollar and a half ? What part of a dollar would a pint cost ?

57. At five dollars a yard, what part of a yard of cloth can be bought for three dollars ? what for two and a half dollars ? for one dollar ?

58. A quince and a peach together cost three cents ; how much did each cost, if the quince cost twice as much as the peach ?

59. Two oranges cost five cents ; how much did each cost, if one cost a cent more than the other ?

60. Three times a number less twice the number equals one ; what is the number ?

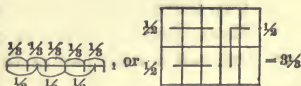
61. Once a number and half the number equal three ; what is the number ?

62. There are three times as many geese in a pond as ducks ; if there are three geese, how many are there altogether ?

63. Sarah has three canary-birds more than Anna ; how many have both, if Anna has one ?

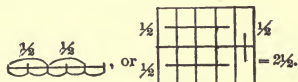
64. In three halves, how many ones ? Three halves equal what part of two ?

65. In five thirds how many ones ? How many halves ?

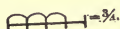


REMARK. — The second part of question No. 65 may be solved with the following diagrams :

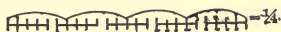
66. In five fourths how many ones? how many halves?
Solution of second by diagrams:



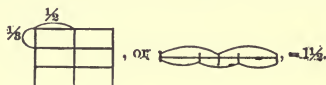
67. One and one half is what part of two? Solution:



68. One and one fourth is what part of five? Solution :



69. How many thirds in one half? Solution :



70. One third is what part of one half? Solution: One third contains two of the three equal parts of one half, and is therefore two thirds of it.



71. How many fourths in one half?

72. One fourth is what part of one half?

73. How many times is one half in two thirds?

Solution: Examining the diagram, we find that one half of it contains three blocks, and two thirds four; the problem, therefore, is reduced to finding the number of times three blocks are found in four blocks.



Ans. $1\frac{1}{3}$.

74. One third is what part of two thirds?

75. Two fourths are what part of three fourths?

76. Three fifths are what part of four fifths?

77. One half is what part of three fourths?

LESSON ON SIX.

a. What the Pupils Must Discover

- | | | | | |
|-------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 1. $5 + 1 =$ | 11. $6 - 2 =$ | 21. $\frac{2}{3}$ of 6 = | 31. $\frac{1}{3}$ of 6 = | 41. $\frac{5}{6}$ of 2 = |
| 2. $1 + 5 =$ | 12. $6 - 5 =$ | 22. $\frac{1}{2}$ of 6 = | 32. $\frac{2}{3}$ of 6 = | 42. $\frac{3}{4}$ of 4 = |
| 3. $4 + 2 =$ | 13. $6 - 6 =$ | 23. $\frac{2}{3}$ of 6 = | 33. $\frac{4}{5}$ of 6 = | 43. $\frac{5}{6}$ of 4 = |
| 4. $2 + 4 =$ | 14. $6 + 1 =$ | 24. $\frac{1}{3}$ of 6 = | 34. $\frac{1}{2}$ of 1 = | 44. $\frac{1}{5}$ of 5 = |
| 5. $3 + 3 =$ | 15. $6 + 2 =$ | 25. $\frac{2}{3}$ of 6 = | 35. $\frac{3}{4}$ of 1 = | 45. $\frac{2}{5}$ of 5 = |
| 6. $3 \times 2 =$ | 16. $6 + 3 =$ | 26. $\frac{3}{4}$ of 6 = | 36. $\frac{5}{6}$ of 1 = | 46. $\frac{3}{4}$ of 6 = |
| 7. $2 \times 3 =$ | 17. $6 + 4 =$ | 27. $\frac{4}{5}$ of 6 = | 37. $\frac{1}{2}$ of 2 = | 47. $\frac{5}{6}$ of 5 = |
| 8. $1 \times 6 =$ | 18. $6 + 5 =$ | 28. $\frac{5}{6}$ of 6 = | 38. $\frac{1}{3}$ of 4 = | |
| 9. $6 - 1 =$ | 19. $6 + 6 =$ | 29. $\frac{1}{2}$ of 6 = | 39. $\frac{1}{3}$ of 5 = | |
| 10. $6 - 3 =$ | 20. $\frac{1}{3}$ of 6 = | 30. $\frac{2}{3}$ of 6 = | 40. $\frac{5}{6}$ of 2 = | |

b. Examples of Seat-work.

- | | | |
|----------------------------|----------------------------|-----------------------|
| 1. five + one = | 8. one $\times$ six = | 15. six + two = |
| 2. one + five = | 9. six - one = | 16. six + three = |
| 3. four + two = | 10. six - three = | 17. six + four = |
| 4. two + four = | 11. six - two = | 18. six + five = |
| 5. three + three = | 12. six - five = | 19. six + six = |
| 6. three $\times$ two = | 13. six - six = | 20. one half of six = |
| 7. two $\times$ three = | 14. six + one = | |
| 21. two halves of six = | 32. three fifths of six = | |
| 22. one third of six = | 33. four fifths of six = | |
| 23. two thirds of six = | 34. one sixth of one = | |
| 24. one sixth of six = | 35. three sixths of one = | |
| 25. two thirds of six = | 36. four sixths of one = | |
| 26. three thirds of six = | 37. one sixth of two = | |
| 27. four sixths of six = | 38. one sixth of three = | |
| 28. five sixths of six = | 39. one sixth of four = | |
| 29. one fourth of six = | 40. one sixth of five = | |
| 30. three fourths of six = | 41. three sixths of five = | |
| 31. one fifth of six = | 42. five sixths of five = | |
- etc., etc.

c. Suggestive Questions and Problems.

1. Take five cubes. Take another. How many alto-

gether have you taken? (You have taken six.) Five and one are how many?

2. Four and two are how many? Three and two?
3. Three and three are how many ones? how many twos?
4. Six ones are how many twos? how many threes?
5. How many fours in six?
6. Make six of ones? how many does it take?
7. Make six of twos? how many does it take?
8. Make six of threes? how many does it take?
9. Make six in all the ways you can mentally, and in every case tell the result.
10. Three twos are how many ones?
11. Two threes are how many ones?
12. Six ones are how many twos? how many threes?
13. I know a number whose third is one; what is it?
14. I think of a number which doubled makes six; what is it?
15. I think of a number whose half added to it makes six; what is it?
16. If to twice a number I add two, it makes six; what is the number?
17. From what number can I take its half and have one left?
18. What number diminished by its third leaves four?
19. If to a certain number I add its fifth, it makes six; what is the number?
20. From what number can I take half of four and have four left?
21. If to a certain number I add a third of three, it makes two; what is the number?
22. Three times a number less once the number is four; what is the number?
23. What equal numbers make six? What unequal numbers?

24. How many fourths in a half? in a fourth and a half?

25. How many sixths in one? in a half? 

26. A half and a third equal how many sixths? 

27. A half, a third, and a sixth are how many ones? 

28. Two thirds are how many sixths? Three sixths equal how many thirds? how many halves? 

29. Four sixths equal how many thirds? how many halves?

30. Five sixths equal how many thirds? how many halves?

31. At two cents a yard, how many yards of tape could you buy for six cents?

32. How much money will pay for four apples at a half-cent each? at a fourth of a cent each?

33. If plums sell at a fourth of a cent each, how much would six cost?

34. Sarah can walk two miles an hour and Helen three; how long would it take each to walk to a town six miles distant? If they should start together, how far apart would they be at the end of two hours? If they should start together but go in opposite directions, how far apart would they be in one hour?

35. Alice went to the grocery with six cents; she bought pears with one third of them, cherries with one half of them, and chestnuts with the remainder; how much money did she give for each?

36. If fish-hooks cost a half-cent each, how many can Charles buy for three cents?

37. Henry bought rabbits at a half-dollar each; how many did he get for two dollars?

38. When hickory-nuts sell at four cents a quart, how

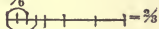
many pints can John get for six cents? How many for two cents?

39. Jennie and Cora went to the cellar for apples; how many did each get, if Cora had twice as many as Jennie, and both had six?

40. If a yard of muslin costs six cents, what does a foot of it cost?

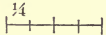
41. One sixth is what part of three sixths? of two sixths? of five sixths?

42. One sixth is what part of one half? Solution by diagram:  $= \frac{1}{3}$.

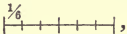
43. One sixth is what part of one fourth? Solution by diagram:  $= \frac{2}{3}$.

44. What part of one is one half of one third? Solution:  one of the blocks, being a half of of a third, is a $\frac{1}{6}$ sixth of one, or of the whole.

45. One half of one half is what part of one?

Solution:  or,  $= \frac{1}{4}$.

46. One third of one half is what part of one?

Solution:  or,  $= \frac{1}{6}$.

47. What is one half of two thirds? of three thirds?

REMARK 1.—It is not deemed necessary to indicate what the pupils should discover, nor to give examples of seat-work, further than the number six.

2.—From here on figures may gradually be introduced.

Suggestive Questions and Problems on Seven.

1. Put four cubes upon the table; put three more on. How many ones have you put on? (Four ones and three ones are seven ones.)

2. Three twos and one are how many?

3. Five and two are how many?

4. Two threes and one are how many?

Seven ones are how many twos? how many threes?

6. What number is three less than seven?

7. What number added to five makes seven?

8. I think of a number whose double and one added make seven; what is it?

9. What number is that whose half added to it makes six?

10. What is one seventh of seven? What are three sevenths of seven? five sevenths? seven sevenths?

11. Two thirds of three apples and three fourths of four apples are how many apples?

12. Jane found one half of four chestnuts and Alvira two thirds of six; how many did both find?

13. Of tape costing a half-cent a yard, Hannah bought one third of three yards, Eva three fourths of four yards, and Emily five sixths of six yards; how much did they all pay?

14. One half of four and two thirds of six are how many ones? how many twos? how many threes?

15. Four sevenths of seven and three fifths of five are how many ones? how many twos? how many threes?

16. Make seven of twos; how many does it take?

17. Make seven of threes; how many does it take?

18. Make seven in all the ways you can mentally, and in each case state the result.

19. How many two-cent postage-stamps could you buy with seven cents?

20. An orange costs three cents and a lemon two thirds as much; how much do both cost?

21. How many three-cent and four-cent pies could you buy with seven cents?

22. How many pints in three quarts? How many gills in a pint and a half?

23. How many feet in two yards? in two-thirds of a yard?

24. If two cakes cost six cents, what is the cost of each?

25. John has six pears which he wishes to share with his two companions; how many will he give to each? What part of them will he keep?

REMARK.—A square has four equal sides and four right angles. When the sides are one inch, it is a square inch; when one foot, a square foot; when a yard, a square yard, etc.

26. How many squares, an inch on each side, could you cut from a piece of paper two inches long and one inch wide?

27. How many squares, two inches on each side, could you make from a piece of paper four inches long and one inch wide?

28. How many squares, a yard on each side, could you cut from a piece of muslin three yards long and two yards wide?

29. Henry borrowed a book from the town library, at a half-cent a week, and kept it until the hire of it amounted to three and a half cents; how long did he keep it?

30. John has two three-cent pieces; how many quarts of milk, at two cents each, can he get for them?

31. Mrs. Smith borrowed Mrs. Brown's sewing-machine, at a quarter of a dollar a week, and kept it a month; how much did she owe Mrs. Brown for the use of it?

32. Mr. Stone hires out his horse at the rate of three dollars for two days; how much would he charge for the use of it one day? How much for three days?

33. If two quarts of milk cost four cents, what will three quarts cost?

Suggestive Questions and Problems on Eight.

1. How many ones in seven? How many twos? How many threes? How many fours? How many fives? How many sixes? How many sevens?

2. Put one to seven and how many have you ? (Seven and one are eight.)
3. Place eight cubes upon the table and divide (separate) them into two equal groups. How many have you in each group ? What part of the whole ?
4. Divide eight cubes into four equal groups and tell me how many you have in each group, also what part of the whole.
5. Divide eight cubes into eight equal groups. How many have you in each ? what part of the whole ?
6. See how many groups of three you can find in eight ; how many of five ; of six ; of seven.
7. Five balls and three balls are how many balls ?
8. Eight less two are how many ?
9. Two and six are how many ? How many more than four ? How many more than seven ?
10. Two fours are how many more than three twos ?
11. One is how much less than one half of two ?
12. What is the difference between three and two thirds of six ?
13. See how many quarts of sand will fill this peck measure.
14. What is the larger measure, a peck or a gallon ? what is the difference ?
15. You may now find how many pecks of sand (or saw-dust) will fill this half-bushel. How many would fill a bushel ? How many half-bushels in a bushel ?
16. How many pints of chestnuts, at four cents a quart, can you buy for six cents ? How many for two cents ?
17. When milk sells at eight cents a gallon, how much would two quarts cost ? How much a pint ?
18. Helen bought a yard and a third of six-cent muslin ; how much did she pay for it ?
19. Margaret bought plums at four cents a quart, how

much did she pay for one fourth of a peck? for one eighth of a peck? for a pint?

20. How many eighths in one half? in one fourth? in two halves? in three fourths?

21. How many more eighths in two fourths than in one half?

22. How many eighths in one half, one fourth, eighth?



23. Which is the larger, one half or one third? What is the difference?

24. Nora bought two thirds of a yard of lace at one store and one half of a yard at another; how many yards altogether did she buy?

25. How many months in eight weeks?

26. Henry was hired to Mr. Carpenter at four dollars a month; how much did he receive for three weeks? for five weeks? for three days? for a week and a half?

27. Mr. Clay hired Mr. Thorne's horse at the rate of eight dollars for six days; he used the horse four days. How much did he owe Mr. Thorne?

28. Joseph borrowed five dollars of Philip, agreeing to pay a half-cent a month for every dollar borrowed; if he kept the money two months, how much did he owe for the use of it?

29. Alfred rented two books from the library; the first was worth one dollar, the second two dollars. If he paid a cent a week for the first and two cents for the second, how much did he owe at the end of two and a half weeks?

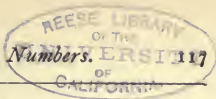
30. One is what part of five? of six? of seven? of eight?

31. Two is what part of six? of seven? of eight?

32. Three is what part of six? of seven? of eight?

33. Four is what part of five? of six? of seven?

34. Five is what part of six? of seven? of eight?



35. Six is what part of seven ? of eight ?
36. Seven is what part of eight ? Seven is how many halves of eight ?
37. If you had one third of two yards of ribbon, what part of a yard could you make of them ?
38. Three fourths of two quarts of peanuts would make how many whole quarts ? how many pints ?
39. The surface of anything—a piece of paper or board, for example—is the outside of it. A square surface an inch on each side and all the angles of which are right angles, is

 called a square inch ; one foot on each side, a square foot, etc. A board three feet long and two feet wide contains how many square feet ?
40. How many square inches could you cut from a piece of paper three inches long and three inches wide ? How many from one four inches long and two inches wide ?
41. How many pieces of paper two inches long and one inch wide could be made from a piece three inches long and two inches wide ?

42. A piece of sheeting one yard long and two thirds of a yard wide contains how many square feet ? what part of a yard ?

43. How many square feet in a piece of cardboard a half-yard square ?

$\frac{1}{2}$	$\frac{1}{4}$
1	$\frac{1}{2}$

REMARK.—A block all of whose surfaces are equal squares is a cube. If the sides of the squares are one inch the block is a cubic inch ; if they are a foot it is a cubic foot, etc.

44. I have a cubical block of wood each of whose edges is two inches in length ; how many cubical blocks whose edges are one inch could be made of it ? How many such could be made of a block two inches long and wide and one inch thick ? How many of one six inches long, one inch wide, and half an inch thick ?

45. A bushel of cloverseed costs four dollars; what is the cost of a peck? of a quart?

46. One fourth is what part of three fourths?

47. Two fifths are what part of three fifths? of four fifths?

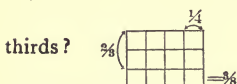
48. One sixth is what part of five sixths? of two sixths?

49. Three sixths are what part of four sixths? of six sixths?

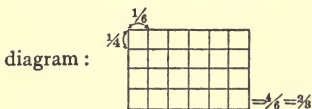
50. Four sevenths are what part of six sevenths? One half is what part of four sevenths?

51. One half is what part of five eighths? Three eighths equal what part of one half?

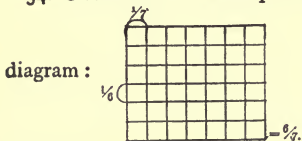
52. One fourth is what part of six eighths? of two



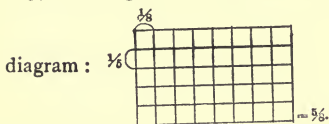
53. One sixth is what part of one fourth? Solution by



54. One seventh is what part of one sixth? Solution by



55. One eighth is what part of one fifth? Solution by



Suggestive Questions and Problems on Nine.

1. Two threes and one two are how many? how many ones?
2. Three twos less two threes are how many?
3. Two fours less two thirds of six are how many? how many twos?
4. Which is the larger, four twos or two fours?
5. Four twos are equal to how many ones?
6. If to eight ones you add one one, how many ones have you? (Eight and one are nine.)
7. How many threes can you find in nine ones, or nine?
8. Nine contains how many twos? fours? fives? sixes? sevens? eights?
9. How can you get one third of nine? What are two thirds of nine?
10. Name the sums of the following numbers as quickly as you can: Three and four; two and six; five and three; two and seven; four and four; one and four; six and three; etc.
11. Name the differences between the following numbers: Three and five; two and six; one and four; two and seven; five and eight; four and nine; one and eight; seven and nine; etc.
12. From eight take two threes; from nine three take twos?
13. I know a number which doubled and one added makes this nine; what is it?
14. Add one third of nine and three fourths of four.
15. Into what like numbers can you divide eight? nine? six?
16. Three fourths of eight and one third of nine equal how many threes?
17. Make nine in all the ways you can mentally, and in each case tell the result.

18. In three yards how many feet?
 19. How many yards in five feet? in eight feet?
 20. What number is that whose third is three?
 21. If to one third of a number I add two, the sum will be three; what is the number?
 22. Three miles added to three fourths of the distance Samuel rode on his bicycle make nine miles; how far did he ride?
 23. If from four times the number of quarts of chestnuts Henry gathered three be subtracted, five will be left; how many did he gather?
 24. Five times the money Sarah received for making a dress added to two dollars equal seven dollars; how much did she receive for her work?
 25. Three fourths of eight cents is two thirds of the price of a gallon of milk; what is the price?
 26. It requires two thirds of nine yards of goods to make Ella a dress, and this is three fourths of the number required to make one for Stella; how many yards does Stella require?
 27. Elmer is twice as old as Carrie, and the sum of their ages is nine years; required the age of each.
 28. From four twos take two thirds of nine, and what is the result?
 29. How many square yards of carpet would be required to cover a floor nine feet long and six feet wide? What would be the cost of the carpet at a dollar and a half a yard?
- REMARK.—The pupils should by this time have learned that the contents of surfaces are obtained by multiplying the length by the breadth, and that the quotient of the contents divided by one of the factors gives the other.
30. I want a piece of paper to cover the bottom of a box that is two inches wide and four and one half inches long; how many square inches must it contain?
 31. The leaves of my book contain eight square inches; how long are they if they are two inches wide?

32. I have a little cubical box whose inside is one inch in length, width, and depth ; how many half-inch cubes would fill it ? How many inch cubes ? Diagram.

33. How many inch cubes would cover the bottom of a box that is three inches long and two inches wide ? How many inch squares would cover it ? How can you ascertain without trying it ?

34. How could you find the number of square inches on the cover of my book ? You may do it.

35. Mr. Fox hired a carriage from the livery stable at the rate of three dollars for six days and used it four days ; how much did he pay for the use of it ?

36. Dora is working for Mrs. Wilson for a dollar and a half a week (of seven days) ; how much will she receive for four weeks and three and a half days ?

37. How many two-cent pieces would you give me for eight cents ? for seven cents ? six cents ? five cents ?

38. As quickly as you can, give me the results of the following : Five less three ; eight less five ; seven less four ; eight less two ; six less three ; nine less six ; nine less four ; seven less two ; three times two ; two times four ; three times three ; four times two ; eight less three ; five and four ; two and six ; twos in four ; threes in six ; fours in eight ; twos in three ; threes in four ; threes in nine ; etc.

39. What part of one is one third of one third. Solu-

tion :

$\frac{1}{9}$		

 = $\frac{1}{9}$.

40. What part of one is one half of one fourth : Solu-

tion :

$\frac{1}{8}$	

 = $\frac{1}{8}$.

41. What part of one is one fourth of one half ? Solve by diagram.

42. One eighth is what part of one fourth? Solve by diagram.

43. Two eighths equal what part of one half. Solve by diagram.

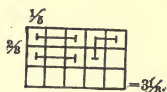
44. Three eighths equal what part of five eighths? Solve by diagram.

45. How many eighths in three fourths? Solve by diagram.

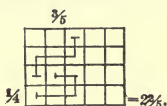
46. How many eighths in two thirds? Solution by diagram.

47. How many thirds in five eighths?

48. How many fifths in two thirds. Solution :

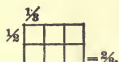


49. How many fourths in three fifths? Solution :



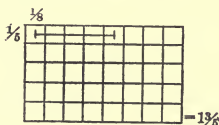
50. How many thirds in four ninths?

51. How many halves in two thirds? Solution :

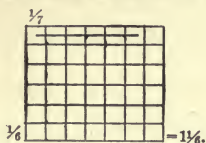


52. One third is what part of one half?

53. How many eighths in one fifth? Solution :



54. How many sevenths in one sixth? Solution:



55. One eighth is what part of one third? Solution:



56. One seventh is what part of one half? Solution:



Suggestive Questions and Problems on Ten.

1. Three threes are how many ones? Add one to them and how many have you? (Nine and one are ten.)
2. Four twos and two ones are how many ones? how many twos?
3. Make ten of fives; how many does it take?
4. Two fours are how many less than ten? how many more than six?
5. Six and how many make ten? Four and how many make ten?
6. How many threes in ten? How many fours? How many sixes?
7. Seven and two are how many? How many more than five? How many less than ten? How many more than eight?
8. How many must be added to two to make ten?
9. Make ten in all the ways you can mentally, and in each case state the result.

10. What is one half of ten? One fifth of ten? Three fifths of ten?

11. To one third of nine add three fourths of eight, and what is the sum?

12. What is the sum of three fourths of eight and two thirds of six?

13. What is the sum of two thirds of three and one half of nine?

14. Ten less seven are how many?

15. How many fifths of five must be added to five sixths of six to make nine?

16. I know a number which doubled makes ten; what is it?

17. I think of a number which taken three times and two added makes eight; what is it?

18. What number diminished by its third leaves six?

19. What number increased by its half equals nine?

20. Six times a number diminished by five equals one; what is it?

21. The sum of two numbers is seven and one of them is four, what is the other?

22. What number added to one third of ten makes four?

23. If you double a number and take four from it you have six; what is the number?

24. Ten thirds equal how many ones?

25. How many ones in ten fourths? in ten fifths?

26. How many tenths in one fifth? in one half?

27. Two fifths and one half equal how many tenths?

28. Can you make tenths of fourths? Why not?

29. What can you make of halves and fourths? Why? of halves and thirds? Why? of halves and fifths?

30. What can you make of halves, thirds, and sixes?

Solution by diagram :



31. One half and two thirds equal how many ones ?
 32. Can you add halves, fourths, and eighths ? How ?
 33. Can you add halves and fifths ? How ? Show by diagram.
-

34. In a piece of goods ten yards long and one yard wide how many square yards ? Prove it.

35. How many yards long is a ten-foot pole ?

36. Helen's apron is two feet in length and a half-yard in width ; what part of a square yard of goods does it contain ?

37. A board eight feet long contains four square feet ; how wide is it ? What would its width be if it contained ten square feet ?

38. My table is a yard in length and five sixths of a yard in width ; how much will it cost to cover it with oilcloth at six tenths of a dollar a square yard ?

39. How would you find the number of square feet of paper required to cover the lower sash of that window ? Do it.

40. Here is a chest that is four feet in length, two feet in width, and one foot in depth ; how many blocks, each containing a cubic foot, would exactly fill it ? How can you tell ? Show by a diagram.

41. Henry bought a half-gallon of cherries at five cents a quart and paid for them with five-cent pieces ; how many did it take ? How many two-cent pieces would pay for them ? How many ten-cent pieces ? How many dimes ?

42. John sold nine pigeons at the rate of a half-dollar a pair ; how much did he receive for them ? How many quarter-dollars ?

43. When sugar is five cents a pound, what will one and cost two thirds pounds ?

44. Mr. Down loans money at a third of a cent on a dol-

lar a month ; how much interest does he receive for the use of five dollars for three and a half months ? for six months ?

45. At a half-cent a month on a dollar, in what time would six dollars give nine cents interest ?

46. At a fourth of a cent a month, how many dollars (how much principal) would in four months give eight and three fourths cents interest ?

47. At what rate (how many cents on the dollar) will four dollars, in three months, give six cents interest ?

48. If Henry adds the interest of nine dollars, at a third of a cent a month, for two and a half months, to the principal, what will the amount (sum) be ?

49. If five oranges cost nine cents, what is the cost of three of them ?

50. When two fifths of a yard of tape cost four cents, what does a half-yard cost ?

51. Three yards of lining cost nine cents ; at the same price, what would two and two thirds yards cost ?

52. At two thirds of a cent each, how many oranges can be bought with two cents ? Solution: 1. If they had cost one third of a cent each, for two cents six could have been bought ; but since they cost *two* thirds of a cent each, only half as many can be bought as if they cost one third of a cent each, or three. 2. Since two thirds of a cent pay for one orange, one third of a cent would pay for one half of it, three thirds for three halves, and two cents for two times three halves, which are six halves, or three.

53. At three fourths of a cent each, how many yards of cord could you buy for two and a half cents ?

54. Mary bought six dollars' worth of cloth at two and a half dollars a yard ; how many yards did she buy ?

55. In ten pecks how many bushels ? In nine pecks how many ?

56. In a half-bushel of cherries, how many gallons ?

57. One is what part of ten ? of nine ? of seven ? of two and a half ?

58. Five is what part of ten ? of nine ? of six ?

59. What part of one is one third of one third ?

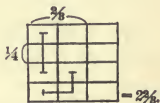


60. What part of one is one fourth of one half ?



61. One half of one fifth is what part of one ? Solve by diagram.

62. How many fourths in two thirds ?



63. How many ninths in one half ?



64. How many eighths in one third ?



65. One fourth is what part of one third ?

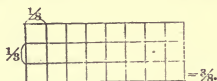


66. One fifth is what part of one half ? of one third ? Solve by diagram.

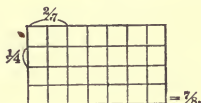
67. How many fifths in one third ? Solve by diagram.

68. One ninth is what part of one third ?

69. One eighth is what part of one third ?



70. One fourth is what part of two sevenths ?



71. One half is what part of three fifths ?

72. How many times can you find three fourths in six ?

-
73. How many times can you find two thirds in five ?
 74. How many times can you find one half in three fourths ?

75. Two thirds are what part of four ?

76. If a tailor can make a pair of pantaloons in a day how long would it take two tailors to make them ?

77. Two boys can pick two quarts of berries in an hour ; how long would it take one of them to pick them ? How long would it take three boys to do it ?

78. Four boys gathered two bushels of chestnuts in two hours ; how many boys could, in the same time, have gathered three bushels ?

79. If five cows can eat an acre of grass in one week, how long would it take one cow to do it ? How long would it take one cow to eat two acres ?

80. Emma can make a dress in two days and Jennie in three ; what part of it can each make in a day ? What part of it could the two together make in a day ? How long would it take them to make it working together ?

81. A man and a boy together do a piece of work in two days ; what part of it does each do, if the man does twice as much as the boy ? How long would it take each alone to do it ?

82. Bessie and Elsie received nine cents for picking strawberries ; how much did each receive, if Elsie received half as much as Bessie ?

83. Henry can do three times as much work in a day as Samuel ; how long would it take Henry to do what Samuel can do in two days ? How long would it take Samuel to do what Henry can do in two days ?

84. Find the length and width of the smallest board that you could exactly cover either with two-inch or three-inch squares.

85. What is the smallest bag that you could make that could be exactly filled with a pint-measure or a quart-measure of chestnuts ?

86. What is the length of the shortest pole that could be exactly measured either with a two-foot measure or with a yard-stick ?

87. What is the smallest number of apples that you could all sell either by twos or threes ?

88. What is the largest measure with which I could empty each of two boxes, one containing a quart of berries, the other a gallon ?

89. Mr. Miller has two baskets of cherries, one containing two quarts, the other three ; what is the largest cup with which he can exactly measure the contents of each basket ?

90. What is the largest measure that is contained in three feet, six feet, and nine feet ? in four feet and eight feet ? in four feet, six feet, eight feet, and ten feet ?

I. *Writing Numbers above Nine.*

REMARK.—The pupils are supposed to have learned to write and use numbers up to ten.

1. They should now be told that nine is the highest number that we can write with one figure, and that above nine the numbers are written as tens and ones.

2. Before the children are taught to write ten, they should have practice in finding the number of tens in a number of objects; tying toothpicks, or other suitable objects, into bundles of tens, affords perhaps the best practice in counting by tens. One bundle should be called one ten, two bundles two tens, three bundles three tens, etc.

3. If a pupil has more toothpicks given him than make an exact number of tens, the remainder should be considered as so many ones (or units). For example, if he should have sixteen given him, after having made all the possible bundles, he would have six toothpicks, or ones, left.

4. To make the bundles, the children should sit or stand around a table, each having a small handful of splints or toothpicks before it, and rubber bands or threads to tie those of a bundle together.

5. After the bundles have been made the children should count both them and the single things or ones, and write the results in columns prepared for the purpose. After each child has written its results or sums in the proper columns, the columns should be added, also the bundles and single things, and the results compared.

Example 1: Suppose there are four children and each has made its bundles; the first having 2 bundles and 4 single things; the second, 4 bundles and 3 single things; the third, 1 bundle and 7 single things; and the fourth, 1 bundle and 2 single things. They now write their results in columns, as here indicated, and add them. The first, or ones' column, gives 1 bundle and 6 single things. Writing the 6 under the ones' column and adding the bundle to the tens, gives as the result of both, 9 bundles and 6 single things.

Tens Ones	
2	4
4	3
1	7
1	2
9	6

After the sums of the columns have been ascertained, the children should give their bundles and remaining toothpicks to one of their number, who, after having made as many bundles as possible of the remaining single toothpicks, should compare her bundles and remaining toothpicks with the sums of the columns.

Example 2: REMARK.—This example, besides carrying the work another step forward, also introduces the nought.

Let us suppose that there are six children in the class, each having made its bundles; the first having 3 bundles and 5 toothpicks; the second, 4 bundles; the third, 2 bundles and 7 toothpicks; the fourth, 3 bundles; the fifth, 2 bundles and 8 toothpicks, and the sixth, 2 bundles. Adding the columns and also the toothpicks, it is found that there are ten bundles and eight bundles; but since there can be no more than ten of a kind, the ten-bundles must be tied together, making a ten-ten bundle, and its number written at the foot of a line next to the left of the tens.

Ten-Tens	Tens	Ones
	3	5
	4	0
	2	7
	3	0
	2	8
	2	0
1	8	0

6. The children should work with toothpicks in connection with figures until they can write and read numbers to one hundred at least. They should also be led to see that all numbers above ten are composed of tens or tens and ones; and instead of continuing to use the names tens and ones, the usual names should gradually be introduced. Thus, for example, instead of saying one ten and six, two tens and four, etc., they should learn the names sixteen, twenty-four, etc.

REMARK.—The nought, having no numerical value, is used to give the significant figures their proper places.

7. The pupils should be led to see that the value of a figure depends upon the place it occupies in a number; that, in general, every ten of one place makes one of the next to the left, or higher, and also the reverse, namely, that every place to the right is one tenth of the next to the left.

2. *Suggestions on Teaching Numbers up to Twenty.*

The numbers up to twenty at least should be so thoroughly taught that the pupils can instantly give the sum or product (less than twenty) of any two of them ; also the reverse ; and if a sum or product is given and one of the two numbers or factors that compose it, the other should instantly be upon the pupils' lips.

3. *A Device for Oral Addition.*

Two numbers below ten, whose sum exceeds ten, may readily be added by taking the difference between the larger and ten from the smaller, adding it to the larger, and adding the remainder of the smaller to ten, the sum. Example : To find the sum of 6 and 8. The difference between 8 and 10 taken from 6, and added to 8, makes 10; and 4, the remainder of the smaller, added to 10, makes 14.

REMARK.—The teacher should lead the children to discover every device that will enable them to overcome their early difficulties and lighten their labor.

4. *The Four Fundamental Processes Carried on Together.*

It should be borne in mind that addition, subtraction, multiplication, and division are to be carried on together.

5. *Pupils May Construct the Tables.*

To familiarize themselves with the relations of number, the children may construct the tables, but in giving the results in class they must not think of the position of the numbers in the table, but must give them instantly—as far as reasonable, automatically.

6. *Suggestive Exercises for Seat Work.*

REMARK.—0 = what ?

1.	2.	3.	4.
1. $10 \div 1 = 11$	1. $0 + 2 = 12$	1. $1 + 0 = 13$	1. $10 + 4 = 0$
2. $9 + 0 = 11$	2. $12 - 0 = 8$	2. $13 \div 1 = 12$	2. $13 \div 1 = 14$
3. $0 - 1 = 10$	3. $0 + 3 = 12$	3. $10 + 3 = 0$	3. $14 - 0 = 9$
4. $8 + 3 = 0$	4. $12 \div 2 = 10$	4. $13 - 0 = 9$	4. $0 + 2 = 7$
5. $11 - 0 = 9$	5. $0 \times 3 = 12$	5. $0 + 7 = 13$	5. $11 + 0 = 14$

6. $0 \div 1 = 11$	6. $12 - 7 = 0$	6. $13 - 5 = 0$	6. $7 \times 2 = 0$
7. $11 - 4 = 0$	7. $705 = 12$	7. $0 + 0 = 13$	7. $608 = 14$
8. $0 + 5 = 11$	8. $0 \times 0 = 12$	8. $0 - 0 = 3$	8. $0 \times 0 = 14$
9. $7 + 0 = 11$	9. $0 - 0 = 7$	9. $13 - 8 = 0$	9. $0 + 0 = 14$
10. $0 + 0 = 11$	10. $0 + 0 = 12$	10. $1102 = 13$	10. $0 - 0 = 5$
11. $0 - 0 = 6$	11. $0 + 0 = 6$	11. $13 - 0 = 6$	11. $0 + 0 = 2$
etc.	etc.	etc.	etc.

5.	6.	7.	8.
1. $0 + 14 = 15$	1. $0 + 10 = 16$	1. $10 + 7 = 0$	1. $0 - 1 = 17$
2. $15 - 0 = 10$	2. $16 - 0 = 9$	2. $0 - 6 = 11$	2. $1 + 0 = 18$
3. $906 = 15$	3. $0 + 3 = 16$	3. $1708 = 9$	3. $1803 = 15$
4. $5 \times 0 = 15$	4. $16 - 10 = 0$	4. $15 + 0 = 17$	4. $15 + 3 = 0$
5. $15 - 10 = 0$	5. $0 + 11 = 16$	5. $17 - 3 = 0$	5. $0 - 5 = 13$
6. $1503 = 5$	6. $16013 = 3$	6. $0 - 14 = 3$	6. $6 + 0 = 18$
7. $0 - 1 = 14$	7. $0 + 0 = 16$	7. $17012 = 5$	7. $18 - 7 = 0$
8. $0 \times 0 = 15$	8. $0 \times 0 = 16$	8. $7 + 0 = 17$	8. $0 + 0 = 18$
9. $0 + 0 = 15$	9. $0 - 0 = 5$	9. $0 + 9 = 17$	9. $0 \times 0 = 18$
10. $0 + 0 = 3$	10. $0 + 0 = 4$	10. $0 + 0 = 17$	10. $0 - 0 = 7$
11. $0 - 0 = 4$	11. $8 + 8 = 0$	11. $0 - 0 = 6$	11. $0 + 0 = 2$
etc.	etc.	etc.	etc.

9.	10.	11.	12.
1. $0 + 3 = 19$	1. $19 + 0 = 20$	1. $\frac{1}{2}$ of $4 = 0$	1. $\frac{1}{2}$ of $0 = \frac{2}{2}$
2. $19 - 0 = 15$	2. $20 - 3 = 0$	2. $\frac{2}{4}$ of $8 = 0$	2. $2 \times 0 = 1$
3. $11 + 8 = 0$	3. $0 + 14 = 20$	3. $\frac{1}{3}$ of $6 = 0$	3. 0 of $6 = 4$
4. $1906 = 13$	4. $20011 = 9$	4. $\frac{1}{9}$ of $9 = 0$	4. $0 \times \frac{2}{2} = 2$
5. $0 + 7 = 19$	5. $3 + 0 = 20$	5. $\frac{1}{5}$ of $5 = 0$	5. $\frac{1}{8}$ of $0 = \frac{1}{8}$
6. $19 - 0 = 6$	6. $4 + 16 = 0$	6. $\frac{1}{4}$ of $4 = 0$	6. $\frac{2}{4}$ of $2 = 0$
7. $3 + 16 = 0$	7. $20 - 8 = 0$	7. $\frac{1}{4}$ of $8 = 0$	7. $\frac{7}{8}$ of $3 = 0$
8. $1908 = 11$	8. $0 + 6 = 20$	8. $\frac{2}{3}$ of $9 = 0$	8. $\frac{6}{6}$ of $4 = 0$
9. $0 - 14 = 5$	9. $0 + 0 = 20$	9. $\frac{2}{4}$ of $7 = 0$	9. $\frac{3}{3}$ of $10 = 0$
10. $0 + 0 = 19$	10. $0 \times 0 = 20$	10. $\frac{2}{2}$ of $4 = 0$	10. $\frac{2}{2}$ of $2 = 0$
11. $0 - 0 = 8$	11. $0 + 0 = 5$	11. $\frac{1}{4}$ of $7 = 0$	11. $\frac{1}{6}$ of $6 = 0$
etc.	etc.		

13.	14.	15.	16.
1. $4 = \frac{2}{2}$ of 0	1. $4 \times \frac{1}{4} = 0$	1. $9 = \frac{3}{3}$ of 0	1. $\frac{2}{2}$ of $12 = 0$
2. $5 = \frac{1}{2}$ of 0	2. $0 \times 4 = 5$	2. $8 = \frac{3}{3}$ of 0	2. $\frac{4}{6}$ of $3 = 0$
3. $6 = \frac{2}{4}$ of 0	3. $\frac{2}{4}$ of $0 = 3$	3. $3 = \frac{2}{2}$ of 0	3. $\frac{2}{2}$ of $1\frac{1}{2} = 0$
4. $0 \times \frac{2}{2} = \frac{2}{2}$	4. $\frac{1}{2}$ of $0 = \frac{1}{2}$	4. $2 = 5 \times 0$	4. $\frac{1}{2}$ of $0 = 1$

- | | | | |
|--------------------------------------|--|---|---|
| 5. $\frac{3}{8}$ of 0 = 3 | 5. $0 \times \frac{1}{2} = 2$ | 5. $12 = \frac{6}{7}$ of 0 | 5. $0 \times \frac{6}{7} = \frac{1}{7}$ |
| 6. $8 = \frac{8}{8}$ of 0 | 6. $\frac{1}{8} \times 0 = \frac{1}{8}$ | 6. $\frac{3}{4} = \frac{3}{8}$ of 0 | 6. $\frac{7}{8}$ of 0 = 7 |
| 7. $9 = \frac{3}{5}$ of 0 | 7. $\frac{5}{6}$ of 0 = $\frac{1}{2}$ | 7. $5 = 3 \times 0$ | 7. $\frac{1}{2}$ of 0 = $3\frac{1}{2}$ |
| 8. $2 = \frac{3}{4}$ of 0 | 8. $0 \times 2 = 3$ | 8. $\frac{1}{4} = \frac{1}{2} \times 0$ | 8. $\frac{1}{4}$ of 2 = 0 |
| 9. $1 = 3 \times 0$ | 9. $2 \times 0 = 5$ | 9. $1 - 0 = \frac{1}{8}$ | 9. $2 \times 0 = \frac{1}{4}$ |
| 10. $6 = \frac{4}{5}$ of 0 | 10. $\frac{5}{6}$ of 5 = 0 | 10. $0 + 3\frac{1}{2} = 5\frac{1}{2}$ | 10. $0 \times 1\frac{1}{4} = 4$ |
| 11. $\frac{3}{8} = \frac{1}{2}$ of 0 | 11. $0 \times \frac{4}{5} = \frac{3}{8}$ | 11. $\frac{2}{5} \text{ of } 5 = 2$ | 11. $\frac{1}{2}$ of 0 = $\frac{3}{4}$ |

17.

1. $\frac{1}{4}$ of 0 = $\frac{1}{2}$
2. $1\frac{1}{8} \times 0 = 4$
3. $0 \times 2 = 1\frac{3}{5}$
4. $\frac{7}{8}$ of 0 = 14
5. $0 \div 4 = \frac{1}{2}$
6. $1\frac{1}{5} \times 0 = 5$
7. $\frac{6}{7} \times 0 = 3$
8. $1\frac{1}{4} \times 0 = 1$
9. $2\frac{1}{2} \times 0 = \frac{1}{2}$
10. $3\frac{1}{2} = \frac{1}{4}$ of 0
11. $2\frac{1}{8} \times 0 = \frac{1}{2}$

18.

1. $\frac{1}{4}$ of $\frac{1}{2} = 0$
2. $\frac{3}{8}$ of $\frac{8}{8} = 0$
3. $\frac{1}{8}$ of 1 = 0
4. $\frac{3}{8}$ of $4 = \frac{1}{2}$ of 0
5. $\frac{7}{8}$ of $5 = 2 \times 0$
6. $1\frac{1}{2} \times 2\frac{1}{8} = \frac{1}{5}$ of 0
7. $\frac{1}{2} = \frac{3}{8}$ of 0
8. $\frac{1}{2}$ of $\frac{3}{4} = 0$
9. $\frac{1}{8} = \frac{3}{8}$ of 0
10. $\frac{1}{5} = \frac{1}{2}$ of 0
11. $\frac{3}{4} = \frac{1}{4}$ of 0

19.

1. $5 = \frac{3}{4}$ of 0
2. $\frac{3}{8}$ of 2 = 4×0
3. $\frac{3}{4}$ of 3 = 3×0
4. $18 = 1\frac{1}{2} \times 0$
5. $2 \times 0 = \frac{1}{2}$ of 3
6. $\frac{1}{2}$ of $\frac{3}{4} = \frac{1}{8}$ of 0
7. $10 = 2 \times \frac{1}{2}$ of 0
8. $\frac{4}{5} = \frac{5}{6}$ of 0
9. $\frac{3}{8}$ of $\frac{1}{5} = \frac{1}{8}$ of 0
10. $\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{18}$ of 0
11. $18 = 4\frac{1}{2} \times 0$

20.

1. $9 = 9 \times \frac{1}{4}$ of 0
2. $\frac{4}{5}$ of $2 = \frac{6}{7}$ of 0
3. $2\frac{1}{2} \times 0 = 18$
4. $0 \times \frac{5}{6} = 15$
5. $\frac{3}{6}$ of $2 = \frac{1}{2} \times 0$
6. $\frac{3}{8}$ of $2 = \frac{1}{4}$ of 0
7. $\frac{1}{2}$ of $\frac{3}{8} = \frac{1}{5}$ of 0
8. $\frac{3}{8}$ of $\frac{1}{2} = \frac{3}{4}$ of 0
9. $\frac{4}{5}$ of $4 = 16 \times 0$
10. $17 = 5\frac{3}{8} \times 0$
11. $\frac{6}{7} \text{ of } 2 = 1\frac{5}{7}$

21.

1. $12 \div 0 = 9$
2. $9 = 12 \times 0$
3. $10 = \frac{5}{6}$ of 0
4. $15 = 16 \times 0$
5. $\frac{1}{2}$ of 5 = 10×0
6. $0 \times 9 = 6$
7. $5 \div 0 = 20$
8. $8 \div 0 = 1\frac{3}{8}$
9. $\frac{3}{8}$ of 0 = 2
10. $6 \times 0 = 1\frac{1}{2}$
11. $\frac{5}{6} \times 0 = \frac{1}{2}$

22.

1. $0 \div 4\frac{1}{8} = 5\frac{1}{2}$
2. $5 = \frac{5}{6}$ of 0
3. $\frac{3}{5}$ of 0 = 9
4. $0 \times 18 = 17$
5. $14 = 15 \times 0$
6. $15 = \frac{7}{8}$ of 0
7. $13 = \frac{3}{8}$ of 0
8. $9 \times 1\frac{1}{2} = 0 \times 2$
9. $3\frac{3}{8} + \frac{5}{6} = 0$
10. $4\frac{1}{7} \div 0 = 3$
11. $\frac{3}{4} \div 0 = \frac{1}{2}$

23.

1. $0 \div 4 = 1\frac{1}{4}$
2. $3 \div 0 = \frac{1}{2}$
3. $6 \text{ of } \frac{3}{8} = 4$
4. $1 \text{ of } 2 = \frac{1}{2}$
5. $\frac{6}{7} \text{ of } 2 = 1\frac{5}{7}$
6. $0 \div \frac{3}{8} = 1$

24.

1. $0 \times 18 = 19$
2. $4 \div 0 = 8$
3. $5 \div \frac{3}{4} = 0$
4. $0 \div \frac{1}{8} = 6$
5. $\frac{3}{4} \div 0 = \frac{7}{8}$
6. $13 \times 0 = 17$

25.

1. $2\frac{1}{2} \div 0 = \frac{1}{2}$
2. $\frac{3}{4} \div 0 = 1\frac{1}{2}$
3. $0 - \frac{5}{6} = \frac{1}{2}$
4. $1\frac{7}{8} \div 0 = 3\frac{1}{2}$
5. $2\frac{1}{3} \times \frac{1}{2} = 0$
6. $1\frac{7}{10} \times 0 = 17$

7. $\frac{3}{4} \div 0 = 3$
 8. $0 - \frac{3}{8} = 2$
 9. $\frac{1}{2} \times \frac{1}{2} = 0$
 10. $2\frac{1}{2} - 0 = 1\frac{1}{2}$
 11. $0 \div 4\frac{1}{2} = 4\frac{1}{2}$

7. $12 \div 0 = 9$
 8. $0 \times \frac{1}{2} = 8\frac{1}{2}$
 9. $9 = 0 \times 12$
 10. $15 \div 0 = 20$
 11. $0 \div \frac{1}{3} = 15$

7. $\frac{3}{4} \div \frac{1}{2} = 3$
 8. $\frac{3}{8} = 5 \times 0$
 9. $2\frac{5}{8} \times 2 = \frac{2}{5}$ of 0
 10. $\frac{9}{10} - 0 = \frac{2}{4}$
 11. $1\frac{1}{8} \div 0 = 3\frac{1}{2}$

26.

1. $0 - 1\frac{3}{4} = 2\frac{3}{8}$
 2. $12 \div 0 = 3\frac{3}{8}$
 3. 0 of $1\frac{1}{8} = 2$
 4. $3 \div 0 = 8$
 5. $4 \div 0 = 3$
 6. $9 \times 0 = 19$
 7. $0 \times 17 = 13$
 8. $7 \div 0 = 12$
 9. $11 \times 0 = 16$
 10. $12 - 0 = 8\frac{1}{2}$
 11. $\frac{3}{8}$ of $\frac{1}{2} = 0$

27.

Lemons. cts. Lemons.

1	2	3
1	3	2
1	1	4
1	4	3
1	2	4
1	5	3
1	2	6
1	6	2
1	4	4
1	5	2

28.

Oranges cts. Oranges.

1	4	5
1	2	7
1	3	5
1	6	3
1	7	2
3	6	2
4	8	3
5	15	2
2	4	3
3	9	4
2	6	3

29.

Apples.	cts.	Apples.
3	12	4
4	16	3
2	10	4
5	15	4
2	4	8
7	7	5
4	20	3
6	12	7
7	14	9
9	18	8
8	16	7

30.

Yds.	cts.	Yds.
4	12	6
6	12	9
8	16	$\frac{1}{2}$
5	15	$\frac{1}{8}$
2	16	$\frac{1}{2}$
$\frac{1}{2}$	2	3
$\frac{1}{8}$	2	$\frac{1}{2}$
1	2 $\frac{1}{2}$	4
$\frac{3}{8}$	8	$\frac{1}{8}$
$\frac{3}{4}$	9	$\frac{3}{8}$
2	16	$\frac{3}{4}$

31.

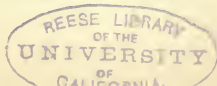
Yds.	cts.	Yds.
$\frac{5}{8}$	15	$\frac{1}{2}$
$\frac{3}{8}$	3	2 $\frac{1}{2}$
1 $\frac{1}{2}$	2	3
2 $\frac{1}{2}$	4	$\frac{5}{8}$
$\frac{3}{4}$	1 $\frac{1}{2}$	$\frac{3}{8}$
$\frac{7}{8}$	1 $\frac{1}{8}$	2 $\frac{1}{2}$
$\frac{5}{8}$	1 $\frac{3}{4}$	4
2	2 $\frac{1}{2}$	$\frac{3}{4}$
$\frac{5}{8}$	2	$\frac{3}{8}$
$\frac{1}{2}$	$\frac{3}{8}$	2 $\frac{1}{2}$
1 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{3}{8}$

REMARK.—To indicate how the exercises under 27–31 should be read, the first under 29 may be taken as an example. If 3 apples cost 12 cents, what cost 4 apples?

7. Suggestive Problems for Oral and Seat Work.

1. If 4 quarts of milk cost 8 cents, what will 8 $\frac{1}{2}$ quarts cost?

2. If 2 $\frac{1}{2}$ yards of tape cost 15 cents, what will 1 $\frac{5}{8}$ yards cost.



3. My table is $3\frac{1}{2}$ feet in length and two feet in width; how many square feet of oilcloth will cover it?

4. A piece of cloth is one yard in length and the same in width; how many square feet of paper would cover it? How many square feet does it contain?

5. How many square feet in a surface a foot square? in one a yard square? in one 3 feet square? in one 4 feet square?

6. How many square yards in a surface 6 feet long and 3 feet wide?

7. I have a box that is 2 feet in length, 1 foot in width, and $1\frac{1}{2}$ feet in height; how many square yards of paper would cover it?

8. Show me an inch on this foot-measure. How many inches in length is the whole measure? How many inches are in a foot? in a $\frac{1}{2}$ foot? in $\frac{2}{3}$ of a foot? in $\frac{3}{4}$ of a foot? in $\frac{5}{8}$ of a foot? in $1\frac{1}{2}$ feet? in $1\frac{3}{4}$ feet?

9. How many feet in 9 inches? in 8 inches? in 14 inches? in 18 inches? in 3 inches?

10. How many quarts in 2 pecks? in a half-bushel?

11. How many pecks in 12 quarts? in 18 quarts? in 6 quarts?

12. At 8 cents a gallon what would 12 quarts of milk cost? What would 18 quarts cost?

13. Take the weights and find out how many ounces make a pound (avoirdupois). How many ounce weights are as heavy as $\frac{3}{8}$ of a pound? What part of a pound weighs as much as 12 ounces?

14. How could you add into one sum $2\frac{1}{2}$ feet and 4 inches? How 5 feet, $\frac{3}{4}$ of a foot, and 8 inches? How 3 yards, $2\frac{2}{3}$ feet and 18 inches?

15. Take the yard-stick and measure $5\frac{1}{2}$ yards from the platform through the middle of this aisle. How many feet did you measure? $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, are called a rod.

16. How many rods in the length of this room? How

many in the width? How many feet in a $\frac{1}{2}$ rod? in $\frac{1}{3}$ of a rod? in $\frac{3}{4}$ of a rod?

REMARK.—The pupils should make inch, foot, yard, and rod measures and use them in measuring lengths and distances. The shorter measures may be made of wood, the longer of twine or cord.

17. Add into one sum 1 rod, $1\frac{1}{2}$ yards, 1 foot, and 8 inches?

18. Find the sum of 3 gallons, 2 quarts, and 1 pint.

19. Find the sum of $\frac{2}{3}$ of a pound and $7\frac{3}{4}$ ounces.

20. How many pints in $\frac{3}{4}$ of a peck, $3\frac{1}{2}$ quarts, and $\frac{2}{3}$ of a pint?

21. How many feet in $\frac{1}{2}$ of a rod, $2\frac{1}{2}$ yards, 2 feet, and 9 inches?

22. How many ounces in $\frac{3}{4}$ of a pound and $2\frac{5}{8}$ ounces?

23. In 15 pints how many gallons? One pint is what part of a gallon? of 3 quarts? Three quarts are what part of three gallons?

24. In 18 inches how many feet? What part of a yard?

25. Nineteen ounces equal how many pounds?

26. Twelve ounces equal what part of a pound?

27. In 17 feet how many yards? how many rods?

28. How many feet in 16 inches? in 10 inches?

29. In 15 weeks how many months? in 17 weeks?

30. How many weeks in $2\frac{1}{2}$ months? in $\frac{1}{3}$ of a month?

31. How many days in $2\frac{1}{2}$ weeks? in $\frac{1}{4}$ of a week?

32. In 14 quarts how many pecks? what part of a bushel?

33. In 20 pints how many pecks? what part of a bushel? how many half-bushels?

34. How many months in $1\frac{1}{2}$ years? in $1\frac{2}{3}$ years? in $\frac{5}{6}$ of a year?

35. How many years in 15 months? in 8 months? in 9 months?

36. Find the interest of $\$2\frac{1}{2}$ at 6 per cent (6c. on a dollar

for a month) for $\frac{2}{3}$ of a year? for 16 months? for 3 months?

37. What is the interest of $\$4\frac{2}{3}$ at 3 per cent for $1\frac{3}{4}$ years? for 4 months?

38. The interest of \$5 for 2 years is 20c.; what is it of \$1 for 1 year?

39. What principal, at 2c. on a dollar a year, will in 16 months give 15c. interest?

40. In what time (how many years) will \$6, at $2\frac{1}{2}$ per cent, give 18c. interest?

41. If by selling my knife for 16c. I gain $\frac{1}{3}$ of the cost, what was the cost?

42. Sarah lost $\frac{1}{3}$ of its cost by selling her bird for 12c.; how much had she paid for it?

43. By selling a book for 10c. I lost $\frac{1}{3}$ of its cost; what should I have sold it for to have gained $\frac{1}{3}$ of its cost?

44. One of two boys can do a piece of work in 3 days, the other in 4 days; what part of it can each do in a day? What part can both together do in a day? How long would it take both together to do the whole of it?

45. If it takes A 2 days to do a piece of work and B 5 days, how many days would it take them together to do it?

46. Four men can do a piece of work in 2 days; how long would it take one of them alone to do it? Solution: $00 + 00 + 00 + 00 = 8$. Let every 0 represent a day's work of each man; then all of them will represent 8 days' work of a man.

47. If 5 girls can make a certain number of dresses in 4 days, in how many days could two of them do the same work? Illustrate by diagram or other form.

48. If 5 boys can pick 30 bushels of apples in a day, how long would it take 1 boy to do it? How long would it require 3 boys to do it?

49. If 10 is $\frac{2}{3}$ of a number, what are $\frac{3}{4}$ of it?

50. If $\frac{5}{8}$ of Henry's ducks equal $\frac{1}{2}$ of his turkeys, and he has 24 ducks, what is the number of his turkeys?

51. A watch and chain cost \$15; what was the cost of each, provided the chain cost $\frac{2}{3}$ as much as the watch?

52. If Jennie adds 8 years to $\frac{1}{3}$ of her age, the sum will be her age; how old is she?

53. One half, $\frac{1}{3}$, and $\frac{1}{4}$ of a certain number added to 8 make 21; what is the number?

54. At $\frac{2}{3}$ of a cent each, how many lemons can be bought for 4 cents?

55. Alberta bought 4 oranges for 3 cents; what was the cost of each? How many did she get for 1 cent?

56. At \$ $\frac{1}{2}$ a yard, how many yards of cloth can be bought for \$6?

57. A boy gave \$3 to a number of beggars, giving to each \$ $\frac{1}{2}$; how many beggars were there?

58. If to Alfred's money you add 4 times his money, he will have 25c.; how much money has he?

59. Find the difference between a square inch and an inch square; between 2 square inches and 2 inches square.

60. What is the difference between a foot square and a half-foot square? Illustrate with diagram.

61. How many 6-inch cubes could you make of a cubic foot? how many 4-inch cubes? How many 2-inch cubes could you make of a 6-inch cube? Diagram.

62. How many cubic feet in a box that is 3 feet long, 2 feet wide, and 1 foot deep?

63. Could you find how many square yards of carpet would cover this floor? How would you do it? If the carpet were $\frac{2}{3}$ of a yard in width, could you find how long a piece it would require to cover the floor? Illustrate with diagram.

64. How could you find the number of square feet of paper required to cover the walls and ceiling of this room? Could you find the number of square yards? How? Could

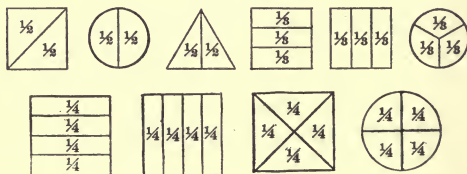
you find how long a roll it would require if the paper were only a half-yard in width ?

65. How many foot cubes would cover the floor of this room ? how many would fill the room ? How many cubic feet in this room ? how many cubic yards ?

8. *Diagrams and Figures.*

1. The teacher should be careful that pupils do not mistake fractional expressions for fractions ; fractions are parts of things or wholes, and fractional expressions, as $\frac{1}{2}$, $\frac{1}{3}$, etc., are the signs or language by means of which they are represented.

2. Fractions may be introduced and to some extent taught by means of folding paper or by various forms of diagrams. The following illustrations, in addition to those already given, may prove helpful to teachers, and may be preferred by some.



3. After pupils have learned to work with real fractions (parts of objects) they should be taught the signs by which they are represented.

9. *Adding, Subtracting, Multiplying, and Dividing by Diagrams.*

1. How many fourths in $\frac{1}{2}$? in $\frac{1}{2}$ and $\frac{1}{4}$? What part of one is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{4} + \frac{1}{4} = ?$ $\frac{1}{2} + \frac{1}{4} = ?$ $\frac{1}{2} + \frac{1}{2} = ?$ $\frac{3}{4} + \frac{1}{4} = ?$ $\frac{1}{2} - \frac{1}{4} = ?$ $\frac{1}{2} \div \frac{1}{4} = ?$ etc.



2. How many sixths in one ? in $\frac{1}{3}$? in $\frac{1}{2}$? What part of

1 is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{6}$ is what part of $\frac{1}{2}$? $\frac{1}{3}$ is what part of $\frac{1}{2}$? $\frac{1}{6}$ is what part of $\frac{1}{3}$? How many sixths in $\frac{1}{3}$? How many $\frac{1}{6}$ in $\frac{2}{3}$? How many $\frac{1}{3}$ in $\frac{1}{2}$? $\frac{1}{6} + \frac{1}{6} = ?$ $\frac{1}{3} + \frac{1}{6} = ?$ $\frac{1}{2} + \frac{1}{6} = ?$ $\frac{2}{3} + \frac{1}{6} = ?$ $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = ?$ $\frac{1}{3} + \frac{1}{3} + \frac{1}{6} = ?$ $\frac{1}{2} + \frac{2}{3} = ?$ $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = ?$ $\frac{2}{3} - \frac{1}{6} = ?$ $\frac{2}{3} - \frac{1}{3} = ?$ $\frac{2}{3} - \frac{1}{2} = ?$ $\frac{1}{2} \div \frac{1}{6} = ?$ $\frac{1}{2} \div \frac{1}{3} = ?$ etc.

$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$

3. How many $\frac{1}{8}$ in 1? in $\frac{1}{4}$? in $\frac{1}{2}$? $\frac{1}{8}$ is what part of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{3}{8}$? $\frac{1}{2}$ of $\frac{7}{8}$? How many $\frac{1}{8}$ in $\frac{3}{4}$? How many $\frac{1}{2}$ in $\frac{5}{8}$? What part of 1 is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{2}$? How many $\frac{1}{4}$ in $\frac{3}{4}$? How many times are $\frac{3}{8}$ contained in $\frac{1}{2}$? $\frac{1}{8} + \frac{1}{8} = ?$ $\frac{1}{4} + \frac{1}{8} = ?$ $\frac{3}{8} + \frac{1}{4} = ?$ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = ?$ $\frac{3}{4} + \frac{3}{8} = ?$ $\frac{1}{4} - \frac{1}{8} = ?$ $\frac{1}{2} - \frac{3}{8} = ?$ $\frac{1}{2} \div \frac{1}{8} = ?$ $\frac{3}{4} \div \frac{3}{8} = ?$ $1 \div \frac{1}{8} = ?$ etc.

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

4. $\frac{1}{3}$ of $\frac{1}{3}$ is what part of 1? How many $\frac{1}{9}$ in $\frac{1}{3}$? $\frac{1}{9}$ is what part of $\frac{1}{3}$? how many $\frac{1}{9}$ in $\frac{1}{2}$? $\frac{1}{3}$ contains how many $\frac{1}{9}$? how many $\frac{1}{9}$ in $\frac{2}{3}$? $\frac{1}{3} + \frac{1}{9} = ?$ $\frac{2}{3} + \frac{1}{3} = ?$ $\frac{1}{3} - \frac{1}{9} = ?$ $\frac{2}{3} - \frac{2}{9} = ?$ $\frac{1}{3} \div \frac{1}{9} = ?$ $1 \div \frac{2}{9} = ?$ etc.

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

5. $\frac{1}{2}$ of $\frac{1}{2}$ is what part of 1? How many $\frac{1}{4}$ in $\frac{1}{2}$? how many $\frac{1}{4}$ in 1? How many $\frac{1}{10}$ in $\frac{1}{2}$? how many $\frac{1}{10}$ in $\frac{1}{3}$? $\frac{1}{3}$ is what part of $\frac{1}{2}$? $\frac{1}{10}$ is what part of $\frac{1}{2}$? How many $\frac{1}{10}$ in $\frac{1}{4}$? How many $\frac{1}{10}$ in $\frac{1}{3}$? How many times is $\frac{1}{10}$ contained in $\frac{3}{10}$? How many times are $\frac{4}{10}$ contained in $\frac{1}{2}$? $\frac{4}{10}$ are what part of $\frac{1}{2}$? $\frac{1}{2} + \frac{2}{5} + \frac{1}{10} = ?$ $\frac{4}{5} + \frac{1}{2} = ?$ $1 - \frac{3}{5} = ?$ $\frac{7}{10} - \frac{2}{5} = ?$ $\frac{1}{2} \div \frac{1}{10} = ?$ $\frac{3}{10} \div \frac{2}{5} = ?$ etc.

$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{10}$	$\frac{3}{10}$

6. $.1 + .1 = ?$ $.2 + .1 = ?$ $\frac{1}{5} + .1 = ?$ $.2 + \frac{1}{2} = ?$ $.4 + .2 = ?$ $.5 - .3 = ?$ $\frac{1}{5} - .4 = ?$ $.4 \div .1 = ?$ $.8 \div \frac{1}{5} = ?$ etc.

$\frac{1}{5}$	$\frac{1}{5}$
$\frac{1}{5}$	.1
$\frac{1}{5}$	.1
$\frac{1}{5}$	.1
$\frac{1}{5}$	.1
$\frac{1}{5}$	.1

7. What is $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{3}$? How many $\frac{1}{12}$ in

11. How many .01 in 1? How many .1 in $\frac{1}{10}$? in $\frac{1}{100}$? How many .1 in .50? $\frac{1}{2} + .1 + .01$

$=? \frac{1}{2} + \frac{1}{10} =? \frac{4}{5} + .3 =? \frac{1}{10} \times \frac{1}{10} =? .1 \times 1 =? .01 \times .1 =? \frac{1}{10} \times .1 =? \frac{1}{4} \times \frac{1}{5} =? \frac{1}{5} \times \frac{1}{10} =? .20 \times .10 =? \frac{1}{100} + .3 =? .1 + .1 =? .1 - .01 =? .1 + \frac{1}{100} =? .1 \div .01 =? \frac{1}{5} \div \frac{1}{100} =? .01 + .4 =? .4 + .07 =? .12 + \frac{9}{100} =? .7 + .6 =? .18 + .24 =? .06 \div .02 =? .50 \div .75 =? .75 \div .3 =? .1\frac{1}{2} + .05 =? .56 - .4 =? .09 - .5 =? .12\frac{1}{2} + .14 =? etc.$

.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	$\frac{1}{10}$

REMARK.—If the foregoing work has been well done, the pupils will, to such an extent, have learned to make their own discoveries—to help themselves—that they will experience comparatively little difficulty with the higher numbers and their combinations.

10. Mental Addition of Two-figured Numbers.

1. Two numbers, each composed of tens and units, may readily be added mentally by first finding the sum of the tens and then adding to it that of the units. *Example*: Find the sum of 28 and 39. Two tens and three tens are five tens, or fifty, and 8 units and 9 units, or 17 units, (added) equal sixty-seven.

2. As soon as the pupils are far enough advanced to do so, instead of saying two tens and three tens are five tens, they should say twenty and thirty are fifty, etc. They should also be led to see that the same combination of units will invariably give the same units' figure. For example, 6 and 7, 16 and 17, 26 and 37, and so on, will all have 3 for the units' figure.

11. Subtracting a Greater from a Less.

1. Subtracting one number from another when some of the figures of the minuend are smaller than those of the

subtrahend to be taken from them, may be developed by

means of tooth-picks. *Example:*
$$\begin{array}{r} 453 \\ -234 \\ \hline 219 \end{array}$$
 Since 4 cannot be taken from 3, one of the ten-bundles must be opened, its contents put to the 3 ones and the 4 taken from the sum.

2. The same thing may also be developed with dollars, dimes, and cents of toy-money. Taking the foregoing example, since 4 cents cannot be taken from 3 cents, one of the ten-cent pieces of the minuend must be exchanged for cents and the latter added to the three to make the subtraction possible. One ten having thus been taken from the 5 of the minuend, 4 remain, from which the 3 of the subtrahend must be taken.

3. The teacher should lead his pupils to see that finding the difference between two numbers is the same as finding what must be added to the smaller to make the larger, and that the difference may therefore be found either by addition or subtraction.

4. Methods of proof should also be developed, so that the pupils may have the means of testing the correctness of their work.

REMARK.—1. As far as possible and as long as necessary the solution of every problem should be illustrated either with objects, diagrams, or drawings. The drawings, at first crude, will with the teacher's assistance gradually improve and will create an interest in that kind of work.

REMARK.—2. It cannot be too strongly impressed upon the mind of the teacher that a part of every lesson should be a review of some of the previously prepared lessons or subjects passed over. Daily reviews give not only clearness to concepts, but impress them firmly upon the mind.

12.

Mental Multiplication.

1. Mentally finding the product of any two numbers up to 100, besides affording a good exercise for the memory, enables the pupils to perform many arithmetical operations

without resorting to pencil and paper. To illustrate the method of doing this when both factors are less than 20, let it be required to obtain the product of 14 by 14. Fourteen is the sum of 10 and 4. First, multiply 14, the multiplicand, by the 10 of the multiplier (or simply annex a nought); next the 10 of the multiplicand by the 4 of the multiplier; add the two products; finally, multiply the 4 of the multiplicand by the 4 of the multiplier and add the product to the sum of the previous products. *Operation:* $14 \times 10 = 140$; $10 \times 4 = 40$; $140 + 40 = 180$; $4 \times 4 = 16$; $180 + 16 = 196$.

2. The foregoing method applies as well to unequal as to equal factors. For example, let it be required to find the product of 19 by 15. *Operation:* $19 \times 10 = 190$; $10 \times 5 = 50$; $190 + 50 = 240$; $9 \times 5 = 45$; $240 + 45 = 285$.

3. To find the product of any two factors that are greater than 20 and less 100, multiply the multiplicand by 10, the product by the number of tens in the tens of the multiplier, the tens of the multiplicand by the units of the multiplier; add product to preceding product; multiply the units of the multiplicand by the units of the multiplier, and add product to preceding sum of products. For example, multiply 48 by 36. *Operation:* $48 \times 10 = 480$; $480 \times 3 = 1440$; $40 \times 3 + 80 \times 3 = 1440$; $40 \times 6 = 240$; $1440 + 200 + 40 = 1680$; $8 \times 6 = 48$; $1680 + 40 + 8 = 1728$.

REMARK.—As an aid to memory, whenever it is possible, round numbers (as in the foregoing operation) should be multiplied and added.

13. *Properties of Nine and Their Application.*

1. If a number is divisible by 9 the sum of its figures is divisible by nine. Illustrative examples: 18, 27, 63, 81, 4896, etc.

2. The remainder, or excess, of the division of a number by 9 is the same as that of the sum of its figures divided by 9. Illustrative example: 5843 divided by 9 leaves 2 as

the excess, and 20, the sum of its figures ($5 + 8 + 4 + 3$), divided by 9, leaves the same.

3. The excess of the division of the sum of two or more numbers by 9 is the same as that of the excess of the sum of their excesses.

From this fact is derived one of the simplest and quickest methods of proving addition and subtraction.

4. *Application to Addition.*—RULE: Find the excess of each of the numbers added, add these excesses, find the excess of their sum, and if the latter is the same as the excess of the sum of the numbers added, the work may be considered correct. Illustrative example:

$$\begin{array}{rcll}
 5873 & \dots\dots 5, & \text{excess of division by 9.} \\
 4652 & \dots\dots 8, & \text{"} & \text{"} & \text{"} \\
 8763 & \dots\dots 6, & \text{"} & \text{"} & \text{"} \\
 579 & \dots\dots 3, & \text{"} & \text{"} & \text{"} \\
 \hline
 19867 & \dots 4. & 22 & \dots 4.
 \end{array}$$

The sum (22) of the excesses, divided by 9, leaves 4 as the excess; and the sum (19867) of the numbers divided by 9 leaves 4, the same excess.

5. *Application to Subtraction.*—RULE: Find the excess of the minuend, subtrahend, and remainder or difference, add those of the remainder and of the subtrahend, and if their sum, or the excess of their sum, is the same as that of the minuend, the work may be considered correct. Illustrative example:

$$\begin{array}{rcl}
 8975 & \dots\dots 2, & \text{excess.} \\
 7487 & \dots\dots 8, & \text{"} \\
 \hline
 1488 & 3, & \text{"}
 \end{array}$$

Adding (3) the excess of the remainder and (8) that of the subtrahend and dividing the sum by nine, the excess found is 2, the same as that of the minuend.

NOTE.—The excess of the division of the product of two or more factors by 9 is the same as the excess of the product of the excesses of the factors.

From this fact is derived one of the simplest and most expeditious modes of proving multiplication and division.

6. *Application to Multiplication.*—RULE: Find the excess of the product and of each of the factors, multiply that of the multiplicand (or the reverse) by that of the multiplier, find the excess of their product, and if it is the same as that of the product of the factors, the work may be regarded as correct. Illustrative example :

$$\begin{array}{r}
 5873 \dots 5 \\
 687 \dots 3 \\
 \hline
 41111 \quad 15 \dots 6 \\
 46984 \\
 35238 \\
 \hline
 4034751 \dots \dots \dots 6
 \end{array}$$

The excess of the product (4034751) is 6, that of the multiplicand (5873) 5, and that of the multiplier 3; and the excess of the product (15) of the latter two is 6, the same as that of the product of the factors.

7. *Application to Division.*—RULE: Find the product of the excess of the divisor and of that of the quotient; to it add the remainder, or its excess (if it has one); and if the excess of the sum is the same as that of the dividend, the quotient may be considered correct. Illustrative example :

$$\begin{array}{r}
 26) 6547 \text{ (251} \\
 \underline{52} \\
 134 \\
 \underline{130} \\
 47 \\
 \underline{26} \\
 21
 \end{array}$$

The excess of 9's of the divisor (26) is 8, that of the

quotient (251), 8, their product is 64 ; to this add (3) the excess of the remainder (21), and the excess of the sum (67) is 4, the same as that of the dividend.

NOTE.--The only case in which the foregoing proofs could fail would be when one error would balance another--when, for instance, 56 would be written 65, or 0 written instead of 9 ; but since such errors are of the rarest occurrence, the proof by the rejection of the 9's may be considered of equal validity with any others.

14. *Composition of Numbers.*

1. The pupils should be led to see how numbers are composed. The following may serve to show how it may be done :

$$78964 = 70000 + 8000 + 900 + 60 + 4.$$

$$9000 + 700 + 80 + 3 = 9783.$$

2. The numbers may also be written in vertical columns :

78964	9000
-----	700
70000	80
8000	3
900	-----
60	9783
4	

15. *Some Points in Multiplication.*

1. When the multiplicand consists of several figures and the multiplier of but one, the pupils are usually instructed to begin at the units' place of the multiplicand and to multiply each of its figures in succession by the multiplier ; but this procedure is looked upon by the pupils as arbitrary. If, however, taking the following example, 456×8 , the multiplicand be separated into 400, 50, and 6, and the pupils told to find the sum of the products of these numbers each multiplied by 8, and then be led to see that the usual method is an abbreviation of this, all thought of arbitrariness will disappear.

2. A development of multiplication like the following throws light upon points frequently not well understood :

$$456 \times 234 = 400 \times 200 + 400 \times 30 + 400 \times 4 + 50 \times 200 + 50 \times 30 + 50 \times 4 + 6 \times 200 + 6 \times 30 + 6 \times 4 =$$

$$400 \times 200 = 80000$$

$$400 \times 30 = 12000$$

$$400 \times 4 = 1600$$

$$50 \times 200 = 10000$$

$$50 \times 30 = 1500$$

$$50 \times 4 = 200$$

$$6 \times 200 = 1200$$

$$6 \times 30 = 180$$

$$6 \times 4 = 24$$

$$456 \times 234 = 106704$$



3. The following will show why the first figure of the product must be written under its multiplier:

$$456 \times 234 = \begin{cases} 456 \times 4 = 1824 \\ 456 \times 30 = 13680 \\ 456 \times 200 = 91200 \end{cases}$$

$$456 \times 234 = 106704$$

4. Various ways in which the partial products may be written:

$$(1) \begin{array}{r} 456 \\ 234 \\ \hline \end{array}$$

$$\begin{array}{r} 1824 \\ 1368 \\ 912 \\ \hline \end{array}$$

$$106704$$

$$(2) \begin{array}{r} 456 \\ 234 \\ \hline \end{array}$$

$$\begin{array}{r} 912 \\ 1824 \\ 1368 \\ \hline \end{array}$$

$$106704$$

$$(3) \begin{array}{r} 456 \\ 234 \\ \hline \end{array}$$

$$\begin{array}{r} 912 \\ 1368 \\ 1824 \\ \hline \end{array}$$

$$105404$$

$$(4) \begin{array}{r} 456 \\ 234 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ 18 \\ 12 \\ 20 \\ 15 \\ 10 \\ 16 \\ 12 \\ 8 \\ \hline \end{array}$$

$$106704$$

5. Multiplication is a short method of finding the sum of a number of repetitions of the same number.

ADDITION.

$$\begin{array}{r}
 4567 \\
 4567 \\
 4567 \\
 4567 \\
 \hline
 18268
 \end{array}$$

MULTIPLICATION.

$$\begin{array}{r}
 4567 \\
 \times 4 \\
 \hline
 18268
 \end{array}$$

16. *Some Points in Division.*

1. In multiplication, the number to be repeated and that denoting the number of repetitions are given to find the sum; in division, the sum and the repeated number are given to find the number of repetitions; or the sum and the number denoting the repetitions may be given to find the repeated number.

2. Division may be regarded as a short method of performing a number of subtractions with the same subtrahend.

SUBTRACTION.

$$\begin{array}{r}
 2193 \\
 243 \text{ (1)} \\
 \hline
 1950 \\
 243 \text{ (2)} \\
 \hline
 1707 \\
 243 \text{ (3)} \\
 \hline
 1464 \\
 243 \text{ (4)} \\
 \hline
 1221 \\
 243 \text{ (5)} \\
 \hline
 978
 \end{array}$$

DIVISION.

$$\begin{array}{r}
 243 \overline{) 2193} \text{ (9)} \\
 \underline{2187} \\
 6
 \end{array}$$

$$\begin{array}{r} 978 \\ 243 \overline{)6} \end{array}$$

$$\begin{array}{r} 735 \\ 243 \overline{)7} \end{array}$$

$$\begin{array}{r} 492 \\ 243 \overline{)8} \end{array}$$

$$\begin{array}{r} 249 \\ 243 \overline{)9} \\ 6 \end{array}$$

3. In multiplication two factors are given to find their product ; in division the product and one of the factors, to find the other.

17. *Important Divisibilities of Numbers.*

1. A divisor of several numbers is a divisor of their sum and difference.

2. A divisor of the sum of two numbers and one of the numbers divides the other.

3. A divisor of a number is a divisor of any multiple of it.

4. A number is not divisible by any number but its factors.

5. Dividing one of the factors of a number divides the number.

6. Multiplying the divisor divides the quotient.

7. Dividing the divisor multiplies the quotient.

18. *Meanings of Division.*

Of the following indicated division, $8 \div 4$, three different cases may be assumed : 1. How many 4's are in 8 ? 2. Four is what part of 8 ? 3. What is $\frac{1}{4}$ of 8 ? Although the three answers are expressed by the figure 2, no two of them represent the same thing.

19.

Long Division.

1. There is no real difference between short division and so-called long division; both aim at the same end and attain it by the same method. In short division most of the work is done mentally, but in long division the larger divisor makes it necessary to write it.

2. A few short-division problems solved by the long-division process forms one of the best introductions to long division.

20.

Greatest Common Measures.

1. The G. C. M. of several numbers is the largest factor common to all of them. It may consist of a prime number or of the product of several prime numbers.

2. When the numbers are small, either of the following methods may be employed to find the G. C. M.:

$$\begin{array}{r}
 3 \overline{) 9 \quad 27 \quad 45 \quad 81} \\
 \underline{3 \quad 3 \quad 9 \quad 15 \quad 27} \\
 1 \quad 3 \quad 5 \quad 9 \\
 \underline{3 \times 3 = 9, \text{ G. C. M.}}
 \end{array}$$

$$\begin{array}{l}
 9 = 3 \times 3 \\
 27 = 3 \times 3 \times 3 \\
 45 = 3 \times 3 \times 5 \\
 81 = 3 \times 3 \times 3 \times 3 \\
 \underline{3 \times 3 = 9, \text{ G. C. M.}}
 \end{array}$$

3. When the numbers are large the method by division must be employed. Example:

$$\begin{array}{r}
 1679 \overline{) 7981} \quad (4 \\
 \underline{6716} \\
 1265 \overline{) 1679} \quad (1 \\
 \underline{1265} \\
 414 \overline{) 1265} \quad (3 \\
 \underline{1242} \\
 23 \overline{) 414} \quad (18 \\
 \underline{23} \\
 184 \\
 \underline{184}
 \end{array}$$

By the following reasoning the method by division may be proved. In the example given, since the smaller of the

two numbers is not an exact divisor of the larger, it is not their G. C. M.; but since (17, 1) the number sought cannot be greater than (1256) the difference of the two numbers, it may be this difference; a trial, however, shows that it is not, that it is not an exact divisor of (1679) the smaller of the two numbers. Continuing the same reasoning, we find that the G. C. M. cannot be greater than (414) the difference between 1265 and 1679, and as a trial shows that it is not this difference, it may be the difference between a multiple of this difference and 1265, and this is found to be correct.

21. *The Least Common Multiple.*

1. The L. C. M. of several numbers is the smallest number that contains each of them as a factor.

2. The following are the two methods of finding the L. C. M.; that by factoring being the more easily explained.

$$\begin{array}{l}
 10 = 2 \times 5 \\
 18 = 2 \times 3 \times 3 \\
 56 = 2 \times 2 \times 2 \times 7 \\
 75 = 3 \times 5 \times 5 \\
 \hline
 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 12600.
 \end{array}$$

$$\begin{array}{r|rrrr}
 2 & 10 & 18 & 56 & 75 \\
 5 & 5 & 9 & 28 & 75 \\
 3 & 1 & 9 & 28 & 15 \\
 \hline
 & 1 & 3 & 28 & 5 \\
 \hline
 & 2 \times 5 \times 3 \times 3 \times 28 \times 5 = 12600.
 \end{array}$$

22. *Fractions.*

Though fractions have received considerable attention in the preceding pages, a few more thoughts concerning them remain to be given.

1. A so-called compound fraction is an indicated multiplication of fractions and not a fraction.

2. What is called a complex fraction is usually an indicated division of a fraction by a fraction.

3. Multiplying a fraction by a fraction is taking such a part of the multiplicand as the divisor is of the unit.

4. Dividing by a fraction is taking the dividend as many times as the divisor is contained in the unit.

REMARK.—From 3 and 4 of the foregoing, we note that, generally speaking, multiplying by a fraction divides, and dividing multiplies.

5. A fraction may be reduced to its lowest terms by factoring both of its terms and cancelling the common factors ; or, when both terms are large, by dividing them by their G. C. D.

6. The following exercises may be used to show that the value of a fraction is not changed by multiplying or dividing both of its terms by the same number.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}, \text{ etc.}; \quad \frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}, \text{ etc.};$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}, \text{ etc.}; \quad \frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25}, \text{ etc.}$$

REMARK.—Exercises like the foregoing may also be used to show that fractions of unlike denominators may be reduced to the same denominator and added or subtracted.

7. Reducing several fractions of different denominators to the same denominator, by multiplying each numerator by all the denominators except its own, and all the denominators together for a new denominator, multiplies both terms of each fraction by the same number.

Illustrative example :

$$\frac{2}{3} + \frac{4}{5} + \frac{6}{7} = \frac{2 \times 5 \times 7}{3 \times 5 \times 7} + \frac{4 \times 3 \times 7}{5 \times 3 \times 7} + \frac{6 \times 3 \times 5}{7 \times 3 \times 5} = \frac{70}{105} + \frac{84}{105} + \frac{90}{105}.$$

Instead of finding the numerators by the preceding method, they may be found by dividing the common

denominator by the denominator of each fraction and multiplying the quotient by the numerator. •

8. The numerator of a fraction may be divided by an integer (whole number) by first multiplying both terms of the fraction by the integer.

Multiplying the denominator of a fraction by an integer divides the fraction by the integer, because it increases the ratio of the numerator to the denominator by the multiplier.

9. No reason can be assigned for the inversion of the divisor in the division of a fraction by a fraction, but its correctness can be shown by means of an analytic explanation or demonstration. For example, let it be required to divide $\frac{1}{3}$ by $\frac{2}{3}$. Explanation: One third is contained in 1 three times, and $\frac{2}{3}$ (being twice as large) one half as often; that is, the number of times $\frac{2}{3}$ are contained in 1; but in $\frac{1}{3}$ they are contained one eighth as often, and in $\frac{1}{3}$ seven times as often as in $\frac{1}{3}$. *Statement of analysis:* $\frac{3}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{1} = \frac{3}{2} \times \frac{1}{3}$. In this statement we see that the divisor has been inverted; and since the same will invariably be the case, we must infer that inverting the divisor and then multiplying give the correct result.

10. The correctness of the foregoing may also be shown by means of a diagram.

23.

Decimals.

1. Decimals are parts of things, or units, in which the division is made according to the scale of tenths. Decimal expressions, though not decimals, will here for convenience be treated as decimals.

2. Decimals, being derived from integers, should be developed from them. This may be done by continuing the division as far below the decimal point as may be thought necessary. The following example will indicate some of the steps that may be taken to introduce the subject:

$$\begin{aligned}
 &1000, 100, 10, 1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}. \quad 1000 + 100 + 10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}. \\
 &\frac{1}{10} = .1; \quad \frac{1}{100} = .01; \quad \frac{1}{1000} = .001. \quad \frac{1}{10} = \frac{100}{1000}; \quad \frac{1}{100} = \frac{10}{1000}. \\
 &.1 + .01 + .001 = .111. \quad \frac{100}{1000} + \frac{10}{1000} + \frac{1}{1000} = \frac{111}{1000}. \\
 &\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} = \frac{100}{1000} + \frac{10}{1000} + \frac{1}{1000} = \frac{111}{1000} = .1 + .01 + .001 = .111.
 \end{aligned}$$

3. The pupils should be led to see or to discover (1) that the point is the distinguishing mark of decimals, and that it is placed before them to separate them and to distinguish them from integers; (2) that as the value of integers decreases from left to right and increases from right to left, so also does that of decimals; (3) that the value of a decimal depends upon its distance from the point; and (4) that for every place a decimal is moved to the left it is multiplied by 10, and for every place it is moved to the right it is divided by 10.

4. Practice should be given in writing and reading decimals until the pupils can readily do either. Decimals should be read as if they were common fractions, the name of the last place to the right given as the denominator. *Example:* In 0.00456, the last place to the right is that of hundred-thousandths, the fraction is therefore 456 hundred-thousandths.

REMARK.—Decimals may be written in four different ways: 1. In words (three hundredths); 2. In the common fractional form ($\frac{3}{100}$); 3. With the per-cent sign (3%); 4. In the usual form (.03).

5. *Place of Point in Multiplication.*—The following will show how the rule for the location of the point may be derived. Taking $.5 \times .4$ as an example, if we discard the points and multiply 5 by 4, we obtain 20 as the product; but the multiplicand is not 5, but $\frac{1}{10}$ of it, hence the product is $\frac{1}{10}$ of 20, or 2.0; and for a similar reason, since the

divisor is not 4, but $\frac{1}{10}$ of it, the last found product is not 2.0, but $\frac{1}{10}$ of it, or .20. An examination of the number of places in the product shows that it is equal to the sum of those in the factors. In the same manner may the rule be derived when the factors contain two or more decimals.

6. *Place of Point in Division.*—If in $.45 \div .5 = .9$, we discard the points and perform the division, we obtain 9 as the quotient; but since the dividend is not 45, but $\frac{1}{100}$ of it, the quotient must be $\frac{1}{100}$ of 9, or .09. This is the quotient obtained by dividing by 5, ten times the divisor, and is therefore $\frac{1}{10}$ of the correct quotient, or .9. Here we observe that the number of decimal places in the quotient is that by which those in the dividend exceed those in the divisor. The same kind of reasoning will discover the rule when both terms contain several decimals.

24.

Analysis.

I. PROPORTION.—The problems usually found under the head of proportion in books on arithmetic can, with more benefit to the pupils, be solved by analysis and cancellation. Simple problems, such as are found on page 128, should at first be given, and their length and difficulty increased as the pupils show themselves able to master them.

Problem 1.—If in 9 days, of 8 hours each, 20 men can build a wall 40 feet long, 2 feet thick, and 6 feet high, how many men would be required to build a similar wall 60 feet long, 3 feet thick, and 5 feet high, in 15 days, of 12 hours each?

REMARK.—Before the analysis of a problem is commenced, a statement of the conditions of the problem should be made. It should also be observed that the first term of the analysis should be of the same kind as the one required.

Statement of Conditions:

M.	d.	hrs.	ft. l.	ft. th.	ft. h.
20	9	8	40	2	6
?	15	12	60	3	5

Statement of Analysis :

$$\frac{20}{1} \times \frac{9}{1} \times \frac{1}{15} \times \frac{8}{1} \times \frac{1}{12} \times \frac{1}{40} \times \frac{60}{1} \times \frac{1}{2} \times \frac{3}{1} \times \frac{1}{6} \times \frac{5}{1} = 15.$$

Analysis, or Explanation.—Since the work can be done in 9 days by 20 men, it would require 9 times as many men to it in 1 day, and $\frac{1}{15}$ as many in 15 days as in 1 day. That is, if they work 8 hours a day ; but 1 hour a day would require 8 times as many men as 8 hours, and 12 hours a day $\frac{1}{12}$ as many as 1 hour. That is, if they make it 40 feet long ; but 1 foot long would require $\frac{1}{40}$ as many men as 40 feet, and 60 feet 60 times as many as 1 foot. That is, provided they make it 2 feet thick ; but 1 foot thick would require one half as many as 2 feet, and 3 feet three times as many as 1 foot. That is, if they make it 6 feet high ; but 1 foot high would require one sixth as many men as 6 feet, and 5 feet five times as many as 1 foot. t

Of every problem twice as many problems can be made as it has terms. Of the following eleven, made from the foregoing, only the statements of the conditions and of the analyses will be given. The problems can be read from the statements.

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 2.</i>	15	15	12	60	3	5
	?	9	8	40	2	6

$$\frac{15}{1} \times \frac{15}{1} \times \frac{1}{9} \times \frac{12}{1} \times \frac{1}{8} \times \frac{60}{1} \times \frac{40}{1} \times \frac{1}{3} \times \frac{2}{1} \times \frac{1}{5} \times \frac{6}{1} = 20.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 3.</i>	20	9	8	40	2	6
	15	?	12	60	3	5

$$\frac{9}{1} \times \frac{20}{1} \times \frac{1}{15} \times \frac{8}{1} \times \frac{1}{12} \times \frac{1}{40} \times \frac{60}{1} \times \frac{1}{2} \times \frac{3}{1} \times \frac{1}{6} \times \frac{5}{1} = 15.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 4.</i>	15	15	12	60	3	5
	20	?	8	40	2	6

$$\frac{15}{1} \times \frac{15}{1} \times \frac{1}{20} \times \frac{12}{1} \times \frac{1}{8} \times \frac{1}{60} \times \frac{40}{1} \times \frac{1}{3} \times \frac{2}{1} \times \frac{1}{5} \times \frac{6}{1} = 9.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 5.</i>	20	9	8	40	2	6
	15	15	?	60	3	5

$$\frac{8}{1} \times \frac{20}{1} \times \frac{1}{15} \times \frac{9}{1} \times \frac{1}{15} \times \frac{1}{40} \times \frac{60}{1} \times \frac{1}{2} \times \frac{3}{1} \times \frac{1}{6} \times \frac{5}{1} = 12.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 6.</i>	15	15	12	60	3	5
	20	9	?	40	2	6

$$\frac{12}{1} \times \frac{15}{1} \times \frac{1}{20} \times \frac{15}{1} \times \frac{1}{9} \times \frac{1}{60} \times \frac{40}{1} \times \frac{1}{3} \times \frac{2}{1} \times \frac{1}{5} \times \frac{6}{1} = 8.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 7.</i>	20	9	8	40	2	6
	15	15	12	?	3	5

$$\frac{40}{1} \times \frac{1}{20} \times \frac{15}{1} \times \frac{1}{9} \times \frac{15}{1} \times \frac{1}{8} \times \frac{12}{1} \times \frac{2}{1} \times \frac{1}{3} \times \frac{6}{1} \times \frac{1}{5} = 60.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 8.</i>	15	15	12	60	3	5
	20	9	8	?	2	6

$$\frac{60}{1} \times \frac{1}{15} \times \frac{20}{1} \times \frac{1}{15} \times \frac{9}{1} \times \frac{1}{12} \times \frac{8}{1} \times \frac{3}{1} \times \frac{1}{2} \times \frac{5}{1} \times \frac{1}{6} = 40.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 9.</i>	20	9	8	40	2	6
	15	15	12	60	?	5

$$\frac{2}{1} \times \frac{1}{20} \times \frac{15}{1} \times \frac{1}{9} \times \frac{15}{1} \times \frac{1}{8} \times \frac{12}{1} \times \frac{40}{1} \times \frac{1}{60} \times \frac{6}{1} \times \frac{1}{5} = 3.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 10.</i>	15	15	12	60	3	5
	20	9	8	40	?	6

$$\frac{3}{1} \times \frac{1}{15} \times \frac{20}{1} \times \frac{1}{15} \times \frac{9}{1} \times \frac{1}{12} \times \frac{8}{1} \times \frac{60}{1} \times \frac{1}{40} \times \frac{5}{1} \times \frac{1}{6} = 2.$$

	M.	d.	hrs.	ft. l.	ft. th.	ft. h.
<i>Problem 11.</i>	20	9	8	40	2	6
	15	15	12	60	3	?

$$\frac{6}{1} \times \frac{1}{20} \times \frac{15}{1} \times \frac{1}{9} \times \frac{15}{1} \times \frac{1}{8} \times \frac{12}{1} \times \frac{40}{1} \times \frac{1}{60} \times \frac{2}{1} \times \frac{1}{3} = 5.$$

3. How many cords of wood in a pile 80 feet long, 12 feet high, and 4 feet thick?

Cords.	ft. l.	ft. h.	ft. th.
1	8	4	4
?	80	12	4

$$\frac{1}{4} \times \frac{1}{8} \times \frac{80}{1} \times \frac{1}{4} \times \frac{12}{1} \times \frac{1}{4} \times \frac{4}{1} = 30.$$

4. How many cubic yards of earth is taken from a ditch 120 feet long, 4 feet wide, and 9 feet deep?

Cu. yd.	ft. l.	ft. w.	ft. d.
1	3	3	3
?	120	4	9

$$\frac{1}{4} \times \frac{1}{3} \times \frac{120}{1} \times \frac{1}{3} \times \frac{4}{1} \times \frac{1}{3} \times \frac{9}{1} = 160.$$

5. How many perches of stone in a wall 36 feet long, 2½ feet thick, and 5 feet high?

Perches.	ft. l.	ft. th.	ft. h.
1	16½	1½	1
?	36	2½	5

$$\frac{1}{4} \times \frac{1}{33} \times \frac{2}{1} \times \frac{36}{1} \times \frac{1}{3} \times \frac{2}{1} \times \frac{1}{2} \times \frac{5}{1} \times \frac{5}{1} = 18.18 +.$$

6. What is the number of bushels of wheat a bin 9 feet long, 5½ feet wide, and 1⅔ feet deep.

	Bu.	ft. l.	ft. w.	ft. d.
(1¼ cu. ft. = 1	1	1¼	1	1
bu., nearly.)	?	9	5½	3⅔

$$\frac{1}{4} \times \frac{1}{5} \times \frac{4}{1} \times \frac{9}{1} \times \frac{1}{3} \times \frac{16}{1} \times \frac{1}{4} \times \frac{15}{1} = 144.$$

7. Required the number of gallons of water in a tank 12 feet long, 3½ feet wide, and 1⅔ feet deep.

	Gal.	ft. l.	ft. w.	ft. d.
(1 cu. foot = 7½ gal-	7½	1	1	1
lons, nearly.)	?	12	3½	1⅔

$$\frac{15}{2} \times \frac{12}{1} \times \frac{1}{3} \times \frac{10}{1} \times \frac{1}{3} \times \frac{10}{1} \times \frac{1}{3} \times \frac{5}{1} = 500.$$

II. SIMPLE INTEREST.—Interest can more readily and intelligently be taught to beginners by stating the number of cents on the dollar than by using the term per cent. Six cents on the dollar, for example, is 6 per cent.

If the time is given in years and months, it may be reduced to the fraction of a year or to months; and if it is given in years, months, and days, it is more conveniently reduced to days, counting 30 days to the month and 12 months to the year. If exact interest is required, 365 days must be taken for a year, and the correct number of days between the dates found.

Problem 1: What is the interest of \$804 dollars, at 6c on the dollar, for 9 years 10 months and 5 days?

P.	int.	time.
\$1	6c	1 yr.
\$804	?	9 yr. 10 mo. 5 d.

$$\frac{6}{1} \times \frac{804}{1} \times \frac{1}{360} \times \frac{3545}{1} = \$475.03.$$

	P.	int.	time.
<i>Problem 2:</i>	\$1	?	1 yr.
	\$804	47503c	9 yr. 10 mo. 5d.

$$\frac{47503}{1} \times \frac{1}{3545} \times \frac{360}{1} \times \frac{1}{804} = 6c.$$

	P.	int.	time.
<i>Problem 3:</i>	\$1	6c	1 yr.
	\$804	47503c	?

$$\frac{360}{1} \times \frac{1}{6} \times \frac{47503}{1} \times \frac{1}{804} = 3545 \text{ days} = 9 \text{ yr. } 10 \text{ mo. } 5d.$$

	P.	int.	time.
<i>Problem 4:</i>	\$1	6c	?
	\$804	47503c	9 yr. 10 mo. 5d.

$$\frac{3545}{1} \times \frac{1}{47503} \times \frac{6}{1} \times \frac{804}{1} = 360d. = 1 \text{ yr.}$$

	P.	int.	time.
<i>Problem 5 :</i>	\$1	6c	1 yr.
	?	47503c	9 yr. 10 mo. 5 d.

$$\frac{1}{1} \times \frac{1}{100} \times \frac{47503}{1} \times \frac{360}{1} \times \frac{1}{3645} = \$804.$$

	P.	int.	time.
	?	6c	1 yr.
<i>Problem 6 :</i>	\$804	47503c	9 yr. 10 mo. 5d.

$$\frac{804}{1} \times \frac{1}{47503} \times \frac{6}{1} \times \frac{3645}{1} \times \frac{1}{360} = \$1.$$

The following problem contains dollars and cents in the principal and a fraction in the rate :

	P.	int.	time.
<i>Problem 7 :</i>	\$1.00	5 $\frac{1}{3}$ c	1 yr.
	\$345.75	?	2 yr. 4 mo. 9d.

$$\frac{1\frac{1}{3}}{3} \times \frac{1}{100} \times \frac{\overset{3.842}{\underset{9}{34.575}}}{1} \times \frac{1}{\underset{3}{360}} \times \frac{283}{1} = \$43.49.$$

Explanation.—The interest of 100 cents, or \$1, is $\frac{1}{3}$ cents, of 1 cent it is $\frac{1}{100}$ as much, and of 34575 cents it is 34575 times as much as of 1 cent. That is for 360 days, or 1 year ; for 1 day it is $\frac{1}{360}$ as much, and for 849 days (2 yr. 4 mo. 9 d.) 819 times as much as for 1 day.

The cancellation can be considerably simplified by dividing both numerator and denominator by 1000 ; that is, by striking out 3 noughts of the denominator and pointing off three places in the principal of the numerator.

By continuing the division of the numerator by the denominator or its factors to as many decimals as may be necessary for accuracy, the whole denominator can generally be cancelled.

III. PERCENTAGE.—The following is one of the simplest methods of introducing percentage, the pupils having learned the numbers at least as far as 100. The teacher

should say that the terms here presented are only other names for the fractions with which they are already familiar. Instead of writing the words per cent., the sign (%) meaning hundredths should be used.

$100\% = \frac{100}{100} = 1$, the whole of anything; $75\% = \frac{75}{100} = \frac{3}{4}$; $50\% = \frac{50}{100} = \frac{1}{2}$; $25\% = \frac{25}{100} = \frac{1}{4}$; $20\% = \frac{20}{100} = \frac{1}{5}$; $10\% = \frac{10}{100} = \frac{1}{10}$; $5\% = \frac{5}{100} = \frac{1}{20}$; $1\% = \frac{1}{100}$; $90\% = \frac{9}{10}$; $80\% = \frac{4}{5}$; $70\% = \frac{7}{10}$; $60\% = \frac{3}{5}$; $40\% = \frac{2}{5}$; $33\frac{1}{3}\% = \frac{1}{3}$; $16\frac{2}{3}\% = \frac{1}{6}$; $12\frac{1}{2}\% = \frac{1}{8}$; and $6\frac{1}{4}\% = \frac{1}{16}$.

Many oral problems should be given to familiarize the pupils with the new terms and with the applications of percentage.

The analytic method of solution applies as well to percentage as to proportion and other similar subjects.

NOTE.—Every number is 100 per cent (or the whole) of itself.

Problem 1: 4 is what % of 5?

	No.	%	
(No. = number.)	5	100	$\frac{100}{1} \times \frac{1}{5} \times \frac{4}{1} = 80$.
	4	?	

Explanation.—Since 5 is 100%, 1 is $\frac{1}{5}$ as many%, and 4, 4 times as many as 1.

Problem 2: 4 is 80% of what number?

	No.	%	
	?	100	$\frac{4}{1} \times \frac{100}{80} \times \frac{100}{1} = 5$.
	4	80	

Explanation.—Since 4 is 80% of the number, 1% of it is $\frac{1}{80}$ of 4, and 100%, or the whole of it, 100 times as many.

Problem 3: What is 80% of 5?

	No.	%	
	5	100	$\frac{5}{1} \times \frac{100}{100} \times \frac{80}{100} = 4$.
	?	80	

Problem 4: If 4 is 80% of some number, what% of it is 5?

No.	%	
5	?	$\frac{80}{1} \times \frac{1}{4} \times \frac{5}{1} = 100.$
4	80	

Problem 5: By selling my cow for \$40 I gained 25% on its cost; what did it cost me?

REMARK.—Adding 25% (the gain) to 100% (the cost) and we have 125 % of the cost.

No.	%	
40	125	$\frac{40}{1} \times \frac{1}{125} \times \frac{100}{1} = 32.$
?	100	

Problem 6: On counting my money I found that \$24 was 20% less than I had when I left home; how much had I spent?

REMARK.—Subtracting 20% from 100%, and we have 80%; what was left.

No.	%	
24	80	$\frac{24}{1} \times \frac{1}{80} \times \frac{100}{1} = 6.$
?	20	

REMARK.—The pupils should, as soon as possible, be led to solve percentage problems by the fractional method.

IV. TRUE DISCOUNT.—In true discount the present worth corresponds to the principal in simple interest, the discount to the interest, and the debt, or sum discounted, to the amount. When the sum to be discounted, the rate, and the time are given, to find the present worth, the problem is the same as having given the amount, rate, and time, to find the principal.

Problem: I owe \$540, due in 4 years without interest, money being worth 5%; what sum would discharge the debt to-day?

Explanation.—As \$540 is the amount of the present worth of the debt due in 4 years at 5%, we must find the amount of \$1 for 4 years at 5 %. The interest of \$1 at 5% for 4 years is 20 cents, and this added to the dollar makes



\$1.20, the amount. Now, considering it as a case in simple interest, and we have given two amounts and the principal of one, to find that of the other. As will be observed, therefore, two operations are required ; the first, to find the amount of \$1 ; the second, to find the present worth. The following are the statements for the second part of the solution :

	P.	amt.
(P. = present worth.)	\$1.00	\$1.20
	?	540
$\frac{100}{1} \times \frac{1}{120} \times \frac{54000}{1} = 450 = \text{required present worth.}$		

V. BANK DISCOUNT.—Bank discount differs from true discount in two respects : 1. In being the interest on the face of the note ; 2. In adding 3 or 4 days of grace to the required time.

REMARK.—The face less the discount equals the proceeds.

Explanation.—Since the proceeds are found by subtracting the interest (discount) from the face of the note, hence, when the proceeds are given and the face value is required, the proceeds of \$1 must be found; then the proceeds of the dollar bear the same relation to the dollar as the given proceeds do to the required face of the note. Here, also, it will be observed, two operations are required : the first, to find the proceeds of one dollar ; the second, to find the required face of the note.

	P.	int.	time.	
I.	\$1.00	6c.	360d.	$\frac{6}{1} \times \frac{1}{360} \times \frac{93}{1} = .0155$, discount on \$1.
	\$1.00	?	93d.	$1.00 - .0155 = .9845$, proceeds of \$1.

	Proceeds.	Face.
	\$0.9845	\$1
	\$600	?
(1)	$\frac{1}{1} \times \frac{1}{.9845} \times \frac{600}{1} = 609,446$	required face.
(2)	$\frac{1}{1} \times \frac{1}{.9845} \times \frac{6000000}{1} = 609,446$	“ “

REMARK.—Number (2) is, perhaps, the simpler operation.

VI. TIME PROBLEMS.—Time relations are best represented by the divisions of horizontal lines.

Problem 1: What time of day is it when the time to midnight is twice the time past noon?

Solution.— $\text{N.} \quad \text{P.} \quad \text{M. N.}$ The time past noon and the time to midnight equal 3 times the time past noon; hence, the time from noon to midnight, or 12 hours, is 3 times the time past noon, and once the time past noon, or the required time, is 4 o'clock P. M.

Problem 2: What time of day is it when the time past noon is $\frac{1}{2}$ of the time to midnight?

Solution.— $\text{N.} \frac{1}{2} \text{P.} \frac{1}{2} \frac{1}{2} \text{M. N.}$ The time to midnight and half the time to midnight, or $\frac{3}{2}$ the time to midnight, equal the time from noon to midnight, or 12 hours; hence, $\frac{1}{2}$ the time to midnight equals $\frac{1}{3}$ of 12 hours, or 4 hours, and $\frac{3}{2}$ of the time to midnight equal 8 hours, and the time is 4 o'clock P.M., the same as that of the previous problem.

Problem 3: Required the time of day when the time past noon equals $\frac{1}{4}$ of the time past midnight.

Solution.— $\text{M. N.} \frac{1}{4} \frac{1}{4} \frac{1}{4} \text{N.} \frac{1}{4} \text{P.}$ Since the time past noon is $\frac{1}{4}$ of the time past midnight, the time before noon, or from midnight to noon, must be $\frac{3}{4}$ of the time past midnight, and this is 12 hours. If $\frac{3}{4}$ of the time past midnight is 12 hours, $\frac{1}{4}$ of it, the time past noon, is $\frac{1}{3}$ of 12 hours, or 4 hours, and the required time is 4 o'clock P.M.

Problem 4: What is the hour of day when the time to noon is $\frac{1}{3}$ of the time to 2 o'clock P.M.?

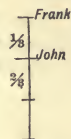
Solution.— $\text{P.} \frac{1}{3} \text{N.} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} 2 \text{ P. M.}$ If the time to noon is $\frac{1}{3}$ of the time to 2 o'clock P.M., then the time past noon, or 2 hours, must be $\frac{2}{3}$ of the time; and if $\frac{2}{3}$ of the time equal two hours, $\frac{1}{3}$ of it, or the time to noon, is $\frac{1}{4}$ of 2 hours, or $\frac{1}{2}$ an hour, and the time required is 11 o'clock A.M.

VII. AGE PROBLEMS.—Age relations are best represented by the divisions of vertical lines.

REMARK.—The difference in the ages of two persons remains the same as long as both of them live. This fact serves as key to the solution of age problems.

Problem 1: John is now 4 years of age and Frank is 10; in how many years will John be $\frac{2}{3}$ as old as Frank?

Solution.—The difference of their ages is 6 years, and this, as the lines show, will be $\frac{1}{3}$ of Frank's age at the required time. As John's age will then be $\frac{2}{3}$ of Frank's, it will be twice 6 years, or 12 years, and since he is now 4, the required time is the difference between 12 and 4, or 8 years.



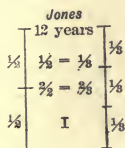
Problem 2: Sarah is 11 years of age and Alice is 20; how long since Sarah was $\frac{1}{2}$ as old as Alice?

Solution.—The difference of their ages, 9 years, was $\frac{1}{2}$ of the age of Alice at the required time; hence Alice was 18 years of age when Sarah was $\frac{1}{2}$ as old; and this was two years ago.



Problem 3: Twelve years ago I was $\frac{1}{2}$ as old as Mr. Jones, now I am $\frac{2}{3}$ as old; what is my present age?

Solution.—Looking at the divisions of the lines, and we find that 12 years ago the difference of their ages was $\frac{1}{2}$ of Jones' age, and now is $\frac{1}{3}$ of it; but as the difference does not change, evidently $\frac{1}{2}$ of Jones' age 12 years ago was $\frac{1}{3}$ of what it now is, and $\frac{2}{3}$ of it then $\frac{2}{3}$ of what it now is. From this we see that 12 years ago Jones was $\frac{2}{3}$ as old as he now is, and since he now is $\frac{3}{2}$, 12 years must be the $\frac{1}{2}$ which he has in these years added. If 12 years equal $\frac{1}{2}$ of Jones' present age, he is now 36, and I am $\frac{2}{3}$ of 36, or 24.



Problem 4: Sarah is now 3 times as old as Martha, but in 9 years will be twice as old; how old is each now?

Solution.—From the divisions of the lines we see that the difference of their ages is now twice Martha's age and in 9 years will be once her age; but as the differences are the same, twice Martha's age now is once what it will be in 9 years, and once her age now is $\frac{1}{2}$ of what it will then be. Since Martha's age will then be $\frac{2}{3}$ of what it now is, she must in 9 years add the other $\frac{1}{2}$; and if 9 years equal $\frac{1}{2}$ of her age then, it must be her present age, and Sarah's is 3 times 9, or 27 years.



REMARK.—If in the foregoing problem the question were, how old will each be then, the following would be the solution, differing only in one point from the preceding:

Solution.—From the divisions of the lines we observe that the difference of their ages is now twice Martha's age, and in 9 years will be once her age; but as the differences are the same, 2 times Martha's age now is once what it will be in 9 years, and once her age now is $\frac{1}{2}$ of what it will then be. Since Martha's age will then be $\frac{2}{3}$ of what it now is, she must in 9 years add the other $\frac{1}{2}$; and if 9 years equal $\frac{1}{2}$ of her age then, $\frac{2}{3}$, or the whole of it, will be 18 years, and Sarah's will be 2 times 18 years, or 36 years.

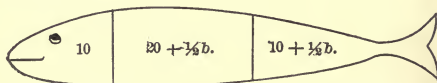
VIII. WATCH AND CHAIN PROBLEMS.—*Problem 1:* A man had 2 watches and only 1 chain. If the chain be put upon the first watch it will make its value twice that of the second; and if it be put upon the second watch it will make its value 3 times that of the first; if the value of the first watch is \$30, what is that of the second, and of the chain?

REMARK.—The equations are designed to indicate the relations of the parts of the problem and thus to aid the solution.

Solution.—As the first watch and chain are together worth twice as much as the second, if to this we add the second we have 3 times the second for the whole, and consequently the second is $\frac{1}{3}$ of the whole. In the second condition we have the second watch and chain worth 3 times as much as the first; if to this we add the first, we have 4 times the first for the whole, and the first is $\frac{1}{4}$ of the whole. Since the first watch is worth \$30, the value of the whole is \$120, that of the second \$40, and $\frac{5}{12}$, that of the chain, \$50.

IX. FISH PROBLEMS.—*Problem*: The head of a fish weighs 10 lbs., the tail weighs as much as the head and $\frac{1}{2}$ the body, and the body weighs as much as the head and the tail; what is the weight of the fish?

Solution.—Make a sketch of the fish, divide it into head, body, and tail, and upon each part place the number given to it in the problem.



The sketch of the body of the fish shows that 20 lbs. and $\frac{1}{2}$ the body is the whole body, but whatever added to a $\frac{1}{2}$ makes the whole must be the other half; therefore 20 lbs. must be the other $\frac{1}{2}$. If $\frac{1}{2}$ the body weighs 20 lbs., the whole of it weighs 40 lbs., and the fish — head 10, body 40, tail 30 = 80 lbs.

X. HOUND-HARE PROBLEMS.—*Problem 1*: A hare is 20 leaps before a hound and takes 4 leaps to the hound's 3, but 3 of the hound's leaps are equal to 6 of the hare's; how many leaps must each take until the hare is caught?

Solution: First make a graphic representation or illustration of the conditions of the problem.

An examination of the accompanying representation of the conditions of the problem shows that while the hound takes 6 of the hare's leaps the hare takes but 4, and thus loses 2. If the hare loses 2 in running 4, to lose 1 it must run $\frac{1}{2}$ of 4, or 2, and to lose 20 it must run 20 times 2, or 40.

When the hound runs 3 it gains the 2 which the hare loses; hence, to gain 1 it must run $\frac{1}{2}$ of 3, or $1\frac{1}{2}$, and to gain 20 it must run 20 times $1\frac{1}{2}$, or 30.

Problem 2: A rabbit is 60 leaps before a hound and takes 9 leaps to the hound's 3, but 2 of the hound's equal 7 of the rabbit's; how many leaps will each take until the rabbit is caught?

Solution.—Make a graphic representation of the conditions, and then, as the numbers of the hound's leaps are unlike, multiply each of the conditions by such a number as shall make them alike, or the same. The remainder of the solution is the same as in the previous problem.

hound.	hare.
3	4
3	= 6

hound.	rabbit.
3	9
2	= 7
6	18
6	= 21

XI. ALLIGATION.—*Problem:* How shall I combine sugars that cost me 6c, 7c, 13c, and 14c a lb. so that I may be able to sell the mixture at 9c a lb?

NOTE.—The only conditions to be observed in the solution of problems of this kind are (1) that the sum of the gains shall equal that of the losses, and (2) that every ingredient upon which there is a gain shall be combined with one upon which there is a loss.

REMARK.—The arrangement of the solution of the problem here given is the usual one, except the horizontal line that separates the losses and gains.

Solution 1.—On a pound of 6c sugar sold for 9c there there will be a gain of 3c, on a pound of 7c sugar sold for 9c there will be a gain of 2c, on a pound of 13c sugar sold for 9c there will be a loss of 4c, and on a

Prices	Diff.	1	2
6	3	5	4
7	2	2	5
13	4	1	3
14	5	3	2

Average
9

pound of 14c sugar sold for 9c there will be a loss of 5c. Now, since the gains and losses must equal each other, if we take 5 pounds of that on which there is a gain of 3c and 3 pounds of that on which there is a loss of 5c, the two will balance each other; and if we take 2 pounds of that on which there is a gain of 2c and 1 pound of that on which there is a loss of 4c, they will balance each other.

Solution 2.—If we take 4 pounds of that on which there is a gain of 3c and 3 pounds of that on which there is a loss of 4c, they will balance each other; and if we take 5 pounds of that on which there is a gain of 2c and 2 pounds of that on which there is a loss of 5c, they will balance each other.

Part X.

GEOGRAPHY.

Introductory Considerations. — Geography is generally classed among the dry subjects, but the dryness is not so much in the subject as it is in the teachers and the teaching. A real teacher can invest any subject with interest, but a lesson-hearer kills even the interest that naturally inheres in a subject.

Much of the matter that has in the past been taught as regularly belonging to geography, and not a little of that still taught to children, is not of the most inviting nature.

No greater mistake can be made in teaching beginners than requiring them to shut their eyes to the world in which they live and to look into a book where all is strange and meaningless ; yet this is the method generally pursued. The children's experiences, which should be used as starting-points, are not only ignored, but regarded as of no value.

Definitions, of whose meaning the children can have no idea, are frequently the food which their mental stomachs are given to digest. After they are supposed to have learned these, then, with their undeveloped imagination, they are expected, from the study of a globe, to form a conception of the earth as a whole, and afterwards to divide it into its so-called members, and these again into sub-members, and so on, until the smallest division has been reached ; this method of procedure being followed, upon the ground that in everything studied or conceived, that is composed of parts, one *must* begin with analysis—with the

whole and go to its parts ; but such a strain at universality, the law "from the whole to its parts," will not bear ; and a little reflection upon the manner in which the mind builds its spacial forms will show the erroneousness of the method.

The suggestions which follow are a departure both in matter and method from what usually passes for geography, but are in harmony with the best in that line of instruction.

The study of geography should be commenced in the primary school, with what the children can observe, and should be largely conversational. Its object should be to awaken an interest on the part of the children in the boundless forms of nature that meet them on every hand.

The work of the primary and others of the lower grades of schools should embrace the following topics :

1. *Land*.—1. Its forms in meadows, uplands, plains, hills, and mountains.

2. The material of which land is composed, namely, soils and rocks.

3. The materials of which the different kinds of soils are composed, namely, loam, sand, gravel, and clay.

REMARK.—The children should examine the different kinds of soil.

4. The part each of the two soils (top and sub) performs in the growth of vegetation.

5. The kinds of soil certain crops demand, the preparation and fertilization of the ground for the reception of the seeds, the time and mode of planting, and care of the plants.

6. The changes which land-forms are undergoing, and their causes.

7. The influence of land-forms upon climate.

NOTE.—The children should make models in sand of the forms of land which they have observed ; this will enable them

later on in their study, when observing a model, to look with their imagination beyond it to that which it represents.

2. *Water*.—1. The forms of water—ice, liquid, and vapor.

2. How ice and vapor are formed and how returned to the liquid state.

NOTE.—Whenever necessary and practicable, processes should be shown by means of experiments.

3. Uses of ice and vapor in nature and to man.

4. The formation of clouds, and their kinds.

5. How rain is produced ; its uses or benefits, especially in the growth of plants.

6. How springs, rivulets, creeks, rivers, ponds, and lakes are formed.

REMARK.—If no pictures are at hand that illustrate it, drawings can be made upon the blackboard to answer the purpose.

7. The uses of water for drinking, washing, cooking, highways, as a force, and as a cooler and moistener of the air.

3. *Air*.—1. The properties of the air.

2. Its use in supporting animal and vegetable life.

3. How winds are caused, and their kinds.

4. The uses of winds in carrying vapor, purifying and cooling the air, propelling vessels, and turning machinery.

5. Changes of temperature, how caused, and how measured.

6. Effects of changes of temperature upon animal and vegetable life.

7. Protection of animal and vegetable life against great changes—summer's heat and winter's cold.

4. *Heat*.—1. Its source or modes of production,

2. Its effect upon different substances,

3. Its uses.

5. *Plants*.—1. Common varieties.

2. The roots, and the part they perform in the growth of plants.

3. The kinds of roots, also duration.

4. Roots used for food.

5. The kinds and use of stem.

6. Leaves, their forms and use.

7. Flowers, their forms, use, and beauty.

REMARK.—The children should be trained to draw and paint plants, including flowers.

8. The kinds of fruit or seeds, when they ripen, and their uses.

9. Buds, what they contain, when they begin to swell, and why they do so.

10. When the blossoms appear.

11. When the fruit ripens.

12. When the leaves begin to fall and what causes them to fall.

13. How roots are protected from the cold in the winter.

14. Food of plants, also cultivation.

15. Medicinal and food plants, also plants that are poisonous.

6. *Domestic Animals*.—1. Varieties, also form or structure.

2. Adaptation of structure to mode of life and subsistence.

3. Grass, grain, and flesh eating animals, and habits and use of each.

4. How fed, sheltered, and cared for.

7. *Wild Animals*.—1. Varieties of form or structure.

2. Adaptation of structure to mode of subsistence and defence or protection.

3. Their habitat, or homes.

4. Grass, grain, nut, and flesh eating quadrupeds.

- (a) Those that are useful for food, skins, and furs.
- (b) How captured, also how tamed.
- 5. Flesh and grain eating birds, and varieties of each.
- (a) How they fly, and how they hold themselves to limbs of trees and other objects.
- (b) How and when they secure their food.
- (c) How they are captured ; also how tamed.
- (a) Which used as food, and why.
- 6. Fish, their varieties, also adaptation of structure to medium in which they live.
- (a) How they move themselves, and adaptation of form to mode of movement.
- (b) Their food, and how they secure it.
- (c) Which are used for food.
- (d) Methods of catching, preserving, and preparing for the table.

Among animals should also be included insects and reptiles, the latter embracing lizards, turtles, tortoises, frogs, toads, and serpents. An examination and study of the nature and habits of these will create an interest in them in the children and will lead them to see that none of God's creatures are useless, but that all of them when properly understood have their purpose, and instead of being our enemies are our friends.

REMARK.—Microscopes are necessary for some of the work referred to.

- 8. *Man*.—1. His superiority to other orders of creation.
- 2. House and home life in comparison with those of other living beings.
- 3. Adaptation to a variety of occupations or employments.
- 4. His genius in using the forces of nature in the performance of labor and in surmounting obstacles.
- 5. Modes of communication and travel.

6. Possibility and means of improvement.
7. Means or modes of enjoyment.
8. Pictures of the various races of the human family should be shown, and the race characteristics described.
9. The effect of climate upon character and disposition should be explained.

REMARK.—As before stated, the foregoing work should, as far as possible, be informal and conversational.

9. *Study from Maps and Models.*—1. After the locality or community in which the children have their homes has been explored and studied as carefully and thoroughly as their age will permit, and a sufficient amount of experiences or apperceiving concepts stored away as constructive material, the children are prepared to extend, their vision, by means of the imagination, to the unseen. This they must learn to do by the use of pictures, maps, and models, and the best and most natural way to learn to understand a map or a model is to help to make one. The school-room is the most convenient and suitable thing to begin with in mapping.

2. To enable the children to judge of distances and areas they should have practice in making measurements. A yard, rod, or mile, either linear or square, should convey something definite to them.

3. The cardinal points—east, west, north, and south—should be determined and marked or fixed upon the floor or elsewhere by means of lines connecting the opposite ones.

4. A map of the school-room may be made upon the floor, but better upon a piece of heavy paper, say a yard square, or larger, if necessary, painted with a mixture of shell-lac (dissolved in alcohol) and lampblack.

5. To make the map, the paper should be laid at some convenient place upon the floor. If the map is to be pro-

portioned to the size of the room—about an inch to a foot—the pupils should make the measurements, determine the proportions, and otherwise assist the teacher in the work.

Frequent questions should be asked of the pupils with regard to the direction of lines, the location of seats, teacher's desk, and other objects.

6. After the map has been made and the pupils can read it—name any object upon it pointed to by the teacher or one of their own number, also its direction from some point named—it **may** be hung upon the wall and again read. The latter reading will prevent the erroneous notion sometimes formed by pupils that north is in a vertical line above south, or at the zenith.

7. After the children have become familiar with the map of the school-room, the surroundings of the school, and, if in a town or city, some of the principal streets and buildings may be added. Next, if it contains enough objects of importance, the county may be drawn, then the state, and other states separately and in sections or groups, until the whole country has been built or mapped, and studied, and the children can in imagination see it or any part of it.

8. As will be noticed, the method here indicated is progressive ; and to show more definitely how it may be carried out, the following suggestions are added : (*a*) If the school is in Pennsylvania, begin with that State ; and if the pupils have had little or no practice in drawing from memory, the teacher should make the first drawing, explaining his work as he proceeds.

(*b*) For the first lesson, the pupils should prepare to draw the outline (of the state) and the rivers, and at the recitation make a sketch of their lesson upon the blackboard, and describe it, the teacher questioning them upon it.

(*c*) For the second lesson, they should add to the first the mountains, principal towns, cities, and other objects of importance. After they have completed their sketches or

maps they should describe them, and the teacher should ask questions about the proportions, the relative position of objects represented upon them, etc.

(*d*) For the third lesson, the second should be reproduced as a review, and the outline and the rivers of New Jersey added.

REMARK 1.—While the pupils are learning the maps, the teacher should, with all the helps at his command, such as models, pictures, and descriptions, enable them, in imagination, to see the states or countries of which the maps are representations.

2.—Descriptions, questions, and reviews of as many previous lessons as may be necessary should constitute a part of every recitation.

(*e*) The fourth lesson should review the third and add to it the mountains, cities, etc., of New Jersey.

(*f*) The fifth lesson should include the fourth and add to it Delaware, with all in it of importance.

(*g*) For the sixth lesson, add to the fifth the outlines of Maryland.

(*h*) For the seventh lesson, add to the sixth whatever may be considered of importance in Maryland.

REMARK.—The daily reviews should be spirited; slow, sleepy work should not be permitted. The pupils should be trained to accurate rapid sketching or mapping.

(*i*) If by this time the pupils have Pennsylvania well pictured in their minds, so that they can make a rapid and sufficiently accurate sketch of it and describe it, they may drop that state for a while and add to the others already drawn Virginia, then West Virginia, and so on, always, in the sketching, dropping those first made as soon as they are well fixed in the mind.

(*j*) A daily review, either oral or by sketching, is a necessity, in order to connect the work from the beginning into a mental picture of the whole and to impress it firmly upon the mind. Frequently all the states that have been studied

should be sketched as a whole. It is of far more importance that what has been learned should find a permanent lodgment in the mind, than that more should be added to what is already fading.

(*k*) After New Jersey has been added to Pennsylvania, instead of taking Delaware next, New York may be taken, then Connecticut, Rhode Island, Massachusetts, Vermont, New Hampshire, and Maine.

REMARK.—If the pupils can do so, they may group two, three, or more states together as a lesson.

(*l*) The teacher should begin with the state in which his school is located and build from that out. He may, of course, begin with some other state, but it is more natural to begin at home.

(*m*) Only things that are important should be found in the sketches.

(*n*) Maps in which no exactness is required should be proportioned by the eye, without construction-lines. However, if a pupil finds it difficult to give the desired shape to his map while preparing his lesson for a blackboard sketch, dividing his paper into rectangles, proportioned as nearly as possible to those made by the parallels and meridians of the maps in his book, will give him all the points he needs for the required form.

(*o*) While the pupils are doing the work here suggested they should also read the descriptive matter found in their books pertaining to the states which they are sketching, and should be questioned upon it by the teacher. Whatever of this matter may be considered of sufficient importance to constitute a permanent possession of the mind should also enter into the daily reviews.

(*p*) Climate and productions are not limited by political divisions, but belong to physical sections or regions, and, unless peculiar to a state, should be taught with the regions to which they belong.

(*q*) Definitions, instead of being the first thing presented to a learner, should generally be the last, and instead of being memorized from a book should as far as possible be drawn from examples or given from a knowledge of the subject.

(*r*) To complete the map of North America, add to the United States the British possessions, Alaska, and Greenland, on the north, and Mexico, Central America, and the West Indies, on the south, grouping all into one picture.

REMARK.—Countries of which our knowledge is limited and, at best, inaccurate, should have only the outline, principal divisions, rivers, mountains, and cities represented on the map.

(*s*) South America may follow North America, then Africa, Europe, Asia, with its surrounding islands, Australia, and the more important islands of the Pacific Ocean.

9. After a whole country has been mapped and impressed upon the minds of the pupils, its prominent or controlling physical features—those upon which the climate, productions, etc., depend—should be studied on a relief map or on a model in sand.

10. The study of the influence of the physical features of a country should be followed by the position of the country upon the globe, also its position with reference to other countries; and its intellectual, moral, and commercial position or standing among the countries of the earth.

11. Countries should also be compared with each other,—their resemblances and contrasts noted,—as indicated in Part II, under Association.

12. When each country has its place assigned upon the globe, and a picture of the whole has been formed in the minds of the pupils, then they are prepared to study it intelligently as a whole, with its lands, waters, motions, forces operative upon it, diversities, and possibilities of life, etc. This is a study of cause and effect, and suitable only for

pupils of sufficient age and mental development to make broad generalizations.

10. *The Sand-box.*—1. Elevations are best represented by means of models of sand, clay, putty, or paper pulp. A box of inch pine-boards, 3 ft. $\times$ 4 ft. $\times$ 3 in., painted on the inside with two or three thick coats of lead paint, placed upon trestles 2 ft. in height and containing about a peck or ten quarts of moulders' sand obtained at a foundry, is not expensive, and should be found in every school in which geography is taught.

2. That the sand may at all times be ready for use, it should, when not needed, be kept in a rounded heap, pounded together to hold the moisture, and once a day, or oftener, sprinkled with as much water with a sprinkling can as will soak in without running off.

11. *Relief-maps.*—1. Instead of purchasing expensive relief-maps, every teacher can, with the assistance of his pupils, make his own maps of paper pulp. The pulp may be made in the following manner: Take white waste paper, (other paper may be used) and tear it into pieces about an inch square, until enough has been prepared to fill a common-sized wooden water-pail or bucket. Pour enough hot water upon the paper to cover it two or three inches deep, and let it remain on it six or eight hours, or overnight. When ready to make the pulp, pour nearly all the water off, leaving only as much as may be necessary for the proper moisture of the pulp. Pour a quart or more (if the pail is nearly full of paper) of flour starch upon the paper, and work or mix it well in with the hands. Now, after pouring half the mass into another vessel,—the reduction being more easily and quickly made by taking a half-pailful at a time,—let each of three boys take in each hand a stick of hard wood about three feet long, three-fourths of an inch thick, and pointed at one end, and, sitting around the pail, drive them down through the mass as rapidly as possible (frequently

stirring the paper up from the bottom), until the desired condition of the pulp has been attained. Treat the other half in the same manner, and when the pulp has all been made, put it into a stone jar or crock, and cover it well, in order that it may hold the moisture until it is wanted to make the maps.

2. To make the maps, take a piece of cardboard about an eighth of an inch in thickness, proportion the map to the one in the book used as a copy, making it two, two and a half, three, or four times the size of the copy. Cut the card at least two inches larger each way than the proposed map, to allow for an inch border all around. Next, draw the map upon the card, and when done, with the fingers put the pulp on, not thicker than an eighth of an inch, pressing it down well to make it adhere to the card. It is best to begin putting on the pulp around the outline, and afterwards to cover the remainder. No pulp should be put upon places intended to represent lakes, nor should rivers be covered with it. Simply press the pulp down on both sides against the river line, but not together, and when it dries it will separate along the line and represent the river.

3. The maps can be made harder and more durable if, when dry, they are given a coat of gum-arabic. If desired, when the gum-arabic is dry the maps may be painted.

4. To make the pulp adhere more firmly to the cardboard, the latter should also be given a coat of mucilage inside of the outline. If this is done the mucilage should be allowed to dry before the pulp is put on.

Part III.

HISTORY.

1. History is a record of human deeds, either of individuals or communities ; and since deeds, measured by moral standards, may be good or bad, the study of history is the study of conduct, the study of morals. Viewed in this light it has an important bearing upon the formation or growth of character.

2. The greater the age a country has attained the larger its accumulation of historical matter at the command of the teacher. In wealth of material, it is true, the lands beyond the ocean surpass us ; but for the lessons that young Americans need to learn, our stores are not only ample, but superior to all others. Our soil has been consecrated to liberty. The heritage which our forefathers have left us is priceless. No other country upon the globe offers such opportunities for persons of ambition and worth to rise from the humblest walks in life to the highest. No law says, " thus far shalt thou go but no further."

3. But those who are soon to become actors in the drama of life must not only be taught to appreciate the value of the freedom handed down to them, but, above all, how to maintain it ; they must be taught that " righteousness " in the individuals " exalts a nation," and that sin is a reproach to any people. That character or worth makes the man must therefore be impressed upon the minds of the children so as to become the ruling principle of their conduct and lives ; and, to accomplish this end, no other branch of study furnishes material equal to that of history.

4. Erroneous notions concerning the influence of certain kinds of matter prevail among teachers. For example, some labor under the delusion that the study of battles and bloodshed cultivates a spirit of patriotism and bravery. They do not distinguish between bloodthirstiness, and patriotism and bravery. Pupils nourished with the former diet—soon eagerly devoured by boys—frequently find the ordinary quiet walks of life too uneventful for “the fires that in them burn,” and start on a career of lawlessness. The matter for class use for children should therefore be selected with care.

5. Children should be taught, and early, too, that might does not make right in the eyes of the civilized world; and that, consequently, decisions rendered by the sword—a relic of barbarous ages—are not to be relied upon as founded upon justice, nor as compatible with Christian civilization.

6. We look to the past also for guidance in the future. For this purpose, however, much of the past is not only unnecessary, but useless; hence teachers of history should exercise more judgment than is usually done in selecting that which has a direct bearing upon the points they are endeavoring to have brought out or discussed. When the pupils have reached a sufficient age and stage of mental development to do so, the selection of the matter should, as far as possible, be left to the exercise of their own judgment, the teacher simply stating the question to be discussed.

REMARK.—A teacher of history should be free from all taint of political bias. A politician or partisan would on all occasions that offered themselves try to influence his pupils to become of his own sort, and thus defeat the object of the study.

7. When not confined to the dry boxes of the subject, such as uneventful administrations and the like, or to the packing of the memory with dates and less important facts,

and when taught intelligently, history is a study of inference, induction, one of the best thought, or logical, subjects, and should not fail to be intensely interesting.

8. A teacher of history should be a good story-teller. He should be able to put dry facts into the form of stories and invest them with interest. Children are fond of stories, hence this is the best way of presenting the subject to them.

9. If the formation of Christian characters is uppermost in the teacher's mind and his pupils are primarians, he can do nothing better than begin with Bible stories, those of the Old Testament, of Abraham, Isaac, Jacob, Joseph, Moses, Joshua, Samuel, Solomon, David, Elisha, Job, Isaiah, Jeremiah, Daniel, and, in the New Testament, Christ, whose life and teachings furnish endless material for building character.

10. Following Bible stories may come the history of our own country, beginning with that of the community—township or county—in which the school is located, bringing in the Indians, then the discovery and settlement of the country by Europeans, the encroachment of the white man upon the hunting-grounds of the Indians, the resulting animosities and strifes, etc. Special stress should be laid upon the events and influences that have controlled our growth and strength as a nation.

11. With whatever matter the teacher may begin, whether Bible stories or the community, it should be given in the form of anecdotes and stories. No books should be used; for there is no surer way to destroy interest in the subject and create a dislike for it than requiring the contents of books to be recited. However, when pupils are old enough to read understandingly they should be encouraged to consult books, and if the instructions they have received from the lips of the teacher have created the proper interest, they will gladly avail themselves of every opportunity to add to their stock of knowledge.

12. The matter for advanced pupils should frequently, if not generally, be given in the form of problems, or subjects for discussion, to give them practice in solving the perplexing problems that will meet them later in life and which confront the citizens of a country like ours.

13. Teachers of history should bear in mind that a good knowledge of their subject of instruction does not necessarily imply a large accumulation of facts, but the ability to use those at command to the best advantage.

14. A teacher of history should have all the appliances in the way of maps, charts, pictures, etc., that are necessary to give his pupils accurate and clear mental pictures of the events that present themselves in their lessons. It is more frequently on account of failures in imagining than of treachery of memory that lessons are unsatisfactorily prepared. The more real the teacher succeeds in making his instructions the better they will be comprehended and remembered.

15. In history fully as much as in anything else, if not more so, daily reviews constitute a necessity. This is the only way to connect the work and to give it a permanent place in the mind.

Part III.

THE HUMAN BODY.

1. Of all the studies pursued in the schools, that relating to the knowledge and care of our bodies is least understood. It is perfectly safe to say that we do not understand how to live well, and the little we pretend to know we disregard.

2. Our food is selected in almost total indifference of what the system craves in kind, proportion, and quantity. Some kinds come too frequently, others the reverse. Again, some that the system does not need, cannot use, and that therefore do harm, are provided, while others that are needed are not supplied. Neither the work performed nor the season is consulted. The same kind is frequently provided for all, whether suitable or unsuitable, whether their stomachs can bear it or not. All are supposed to be alike.

3. Cooking is done by persons who are ignorant of the laws of health ; frequently they do not know that there are such laws ; and, in the preparation, either destroy the most nutritious part of the food or permit it to evaporate.

4. The amount of exercise needed is as little understood as the food required. Either too much is taken or not enough. It is scarcely known, and still less believed, that the human machine, like others not human, may be run too fast or too slow ; neither of which can long be done with impunity. The avenger invariably comes to call a halt.

5. Rest, in the form of sleep, seems to be too much re-

garded as a thing that can be put off from time to time until there is nothing else to do ; but rest is fully as important as food, and while some may take so much as to become tired of it, others, and many of them, do not take enough, especially those who sleep by the clock and not by what they need. Fortunately, no one needs a clock to tell him when he has had enough ; he has his guage within him. Those who claim, as some do, that time does not permit them to sleep until they feel that they have had sufficient, earlier or later pay the penalty for their indiscretion or imprudence.

6. Cleanliness of person is also a thing that needs more attention than it usually receives. Bathing, some think, should be done now and then, say once a week or month, just as there may be time to spare from other duties. But daily bathing, and during warm weather more frequent, is a necessity to comfort and health. The pores of the skin need to be kept open and active.

7. Ventilation is seldom overdone, but in ninety-nine cases in a hundred the reverse. Pure air, if a little cool, is considered dangerous, while the rankest poison thrown off from the lungs and skin is, with the utmost composure and ignorance, time and again returned to the lungs, until almost complete stupor ensues.

8. What are our schools doing to remedy these defects ? What are they doing to teach right living—well living ? Will memorizing the names and number of the bones, teeth, muscles, nerves, etc., usher in the wished-for millennium ? If it is supposed to do so, it has hitherto proved a signal failure ; and as the past has been so will the future be, unless a wiser course be pursued.

9. It is not unfair to ask, how many persons who undertake to give instruction in this all-important subject are competent to do so ? Not a few of them, judging from their frequent ailments, do not know their own bodies well

enough to take care of them, yet do not hesitate to instruct others how to take care of theirs.

10. The *structure* of the body is well enough understood to be taught with success. All that teachers need is to make themselves thoroughly acquainted with it.

11. With regard to hygiene the case is different. All that can be learned from books and teachers in the present state of knowledge of the subject is some very general facts, nothing more. Until we shall have something much more definite than we now possess, and perhaps ever after, every one must study himself to learn what his well-being demands. This study must be inductive, and must determine what, under varying conditions, is healthful and what injurious.

12. Primary pupils should, as far as possible, be taught objectively, without the use of books. One of the best teachers' helps for giving such instruction is "Practical Work in the School-room," Part I—The Human Body, published by A. Lovell & Co., New York. Lessons on The Boy, in "Systematic Science Teaching," a recent work, published by D. Appleton & Co., New York, will be found exceedingly helpful in showing how such and all other science instruction should be given.

13. For work above the primary classes the following order of presenting the subject may be followed: 1. The framework or bones; 2. The muscles; 3. The skin; 4. Digestion; 5. Food; 6. Circulation; 7. Respiration; 8. The nervous system; 9. The senses.

REMARK.—In connection with each part or organ should also be taught, as far as possible or practicable, its care.

14. All unimportant and unnecessary details and scientific terms—a burden to the memory—should be omitted. It is more important that pupils should become interested in the study of their own bodies than that they should know all

about it. The order of presentation should generally be, first the thing, then its name.

15. To teach the human body successfully, either a skeleton and models of the various organs are required or a manikin ; without either of these it is impossible to give the pupils an accurate conception of the forms and position of the internal organs.

Part III.

CIVIL GOVERNMENT.

1. There is no reason why the children in the lower grades of schools should not be made acquainted with the elements of civil government. There is nothing difficult to understand about the subject, and if presented in an intelligent manner it is an interesting one.

2. The first lessons should be about their own community, whether township or borough, and should include the following: 1. What public duties are, and why public rather than private; 2. By what officers the duties are performed; 3. Whether officers are elected or appointed, and by whom, how, when, and for what length of term; 4. How officers are installed, and when; 5. How and by whom paid; 6. Duties each is required to perform; 7. Rights and duties of citizens; 8. How rights are secured and wrongs redressed; 9. What laws are, by whom made, for whose benefit, and why needed; 10. What the laws forbid.

Next should come the government of the county, and then as much of that of the State as the pupils are old enough to understand and to be interested in.

3. No subject should be presented to pupils before they have reached a period of life at which they can be interested in it; hence the study of the general government should be left for the high-school.



Part III.

DRAWING.

Drawing is a mode of expression that manifests itself early in children. Give one a pencil or a piece of crayon and he will try to express something, however crude or unintelligible the performance may be. Since the desire or instinct exists, why not foster it? What the children need at this period is encouragement, help. Hence, whenever anything presents itself in their lessons that admits of representation by lines or colors, let them try to draw it, or paint it; and, when necessary, show them how to improve their work. If this course be pursued, drawing and painting will be taught in the most natural way, and without a special class or period for it.

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